# Collective flow - theory highlights

**Denes Molnar**

**RIKEN/BNL Research Center & Purdue University**

**Joint CATHIE/TECHQM Workshop**

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$O(20)$ **speakers - sorry if I left anyone's favorite slides out**

**Goal: measure QCD matter properties**
**(EOS, transport coefficients)**

**collective flow signatures play a crucial role in this**

Bass

# 3D-Hydro + Micro Model

Theory DUKE

VISCOUS
full 3-d ~~ideal~~ RFD

QGP evolution

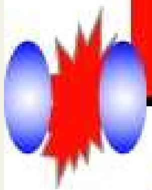
Hadronization

Cooper-Frye formula

Monte Carlo

UrQMD

hadronic rescattering



$T_c$ $T_{SW}$ t fm/c

## Hydrodynamics

- ideally suited for dense systems
  - model early QGP reaction stage
- well defined Equation of State
- parameters:
  - initial conditions
  - Equation of State

+

## micro. transport (UrQMD)

- no equilibrium assumptions
  - model break-up stage
  - calculate freeze-out
  - includes viscosity in hadronic phase
- parameters:
  - (total/partial) cross sections

matching condition:

- use same set of hadronic states for EoS as in UrQMD
- generate hadrons in each cell using local T and $\mu_B$


S.A. Bass & A. Dumitru, Phys. Rev **C61** (2000) 064909
D. Teaney et al, nucl-th/0110037
T. Hirano et al. Phys. Lett. **B636** (2006) 299
C. Nonaka & S.A. Bass, Phys. Rev. **C75** (2006) 014902

# Configurations for initial Run

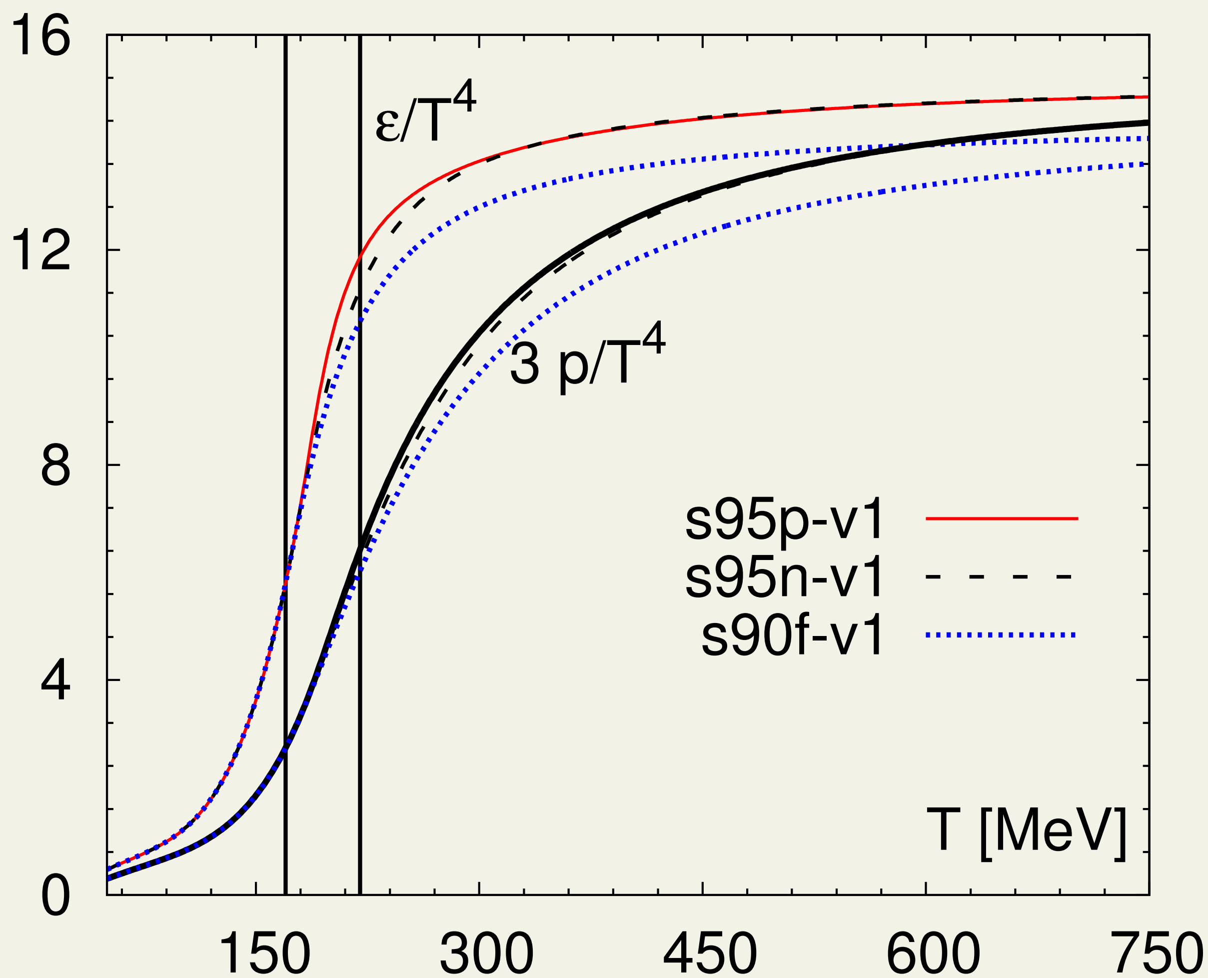
- Fully automate: preFlow+UVH2+UrQMD+CoRAL(HBT)
- Initial set of runs
  - b=5.5 fm (Wounded Nucleon)
  - pre-Flow on/off
  - $T_{initial}$ = 0.35,0.33,0.31 GeV
  - $T_{freeze\text{-}out}$ = 0.15 (CF viscous corrections)
  - $\eta/s$ = 0.08 (0.16, 0.24 next)
  - EOS is vh2 default (full LQCD EOS next)
- No parameters are tuned
  - still checking distributions, last bug-fix was Friday
  - model results unvarnished, data comparisons are scant

## Requires several ingredients:

- knowledge of matter properties Huovinen, Prakash, Bass
- initial conditions Venugopalan, Dumitru, Steinberg
- dissipative hydro results and tests
  Song, Monnai, Denicol, DM, Teaney, El, Niemi
- corroborating observables, such as conical flow Majumder, Neufeld, Xu

# Matter properties

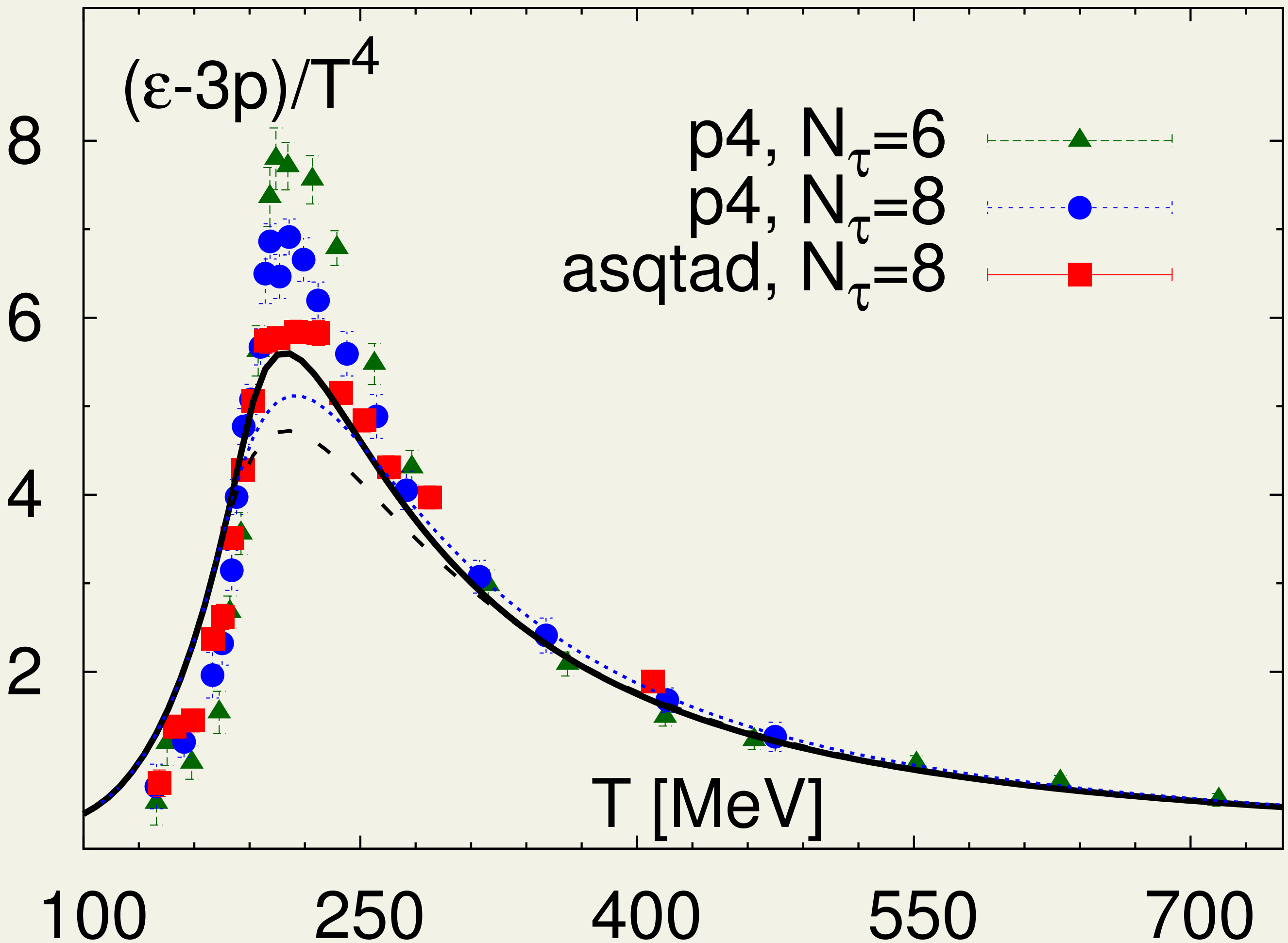
# Phenomenological EoS




- **pressure from**

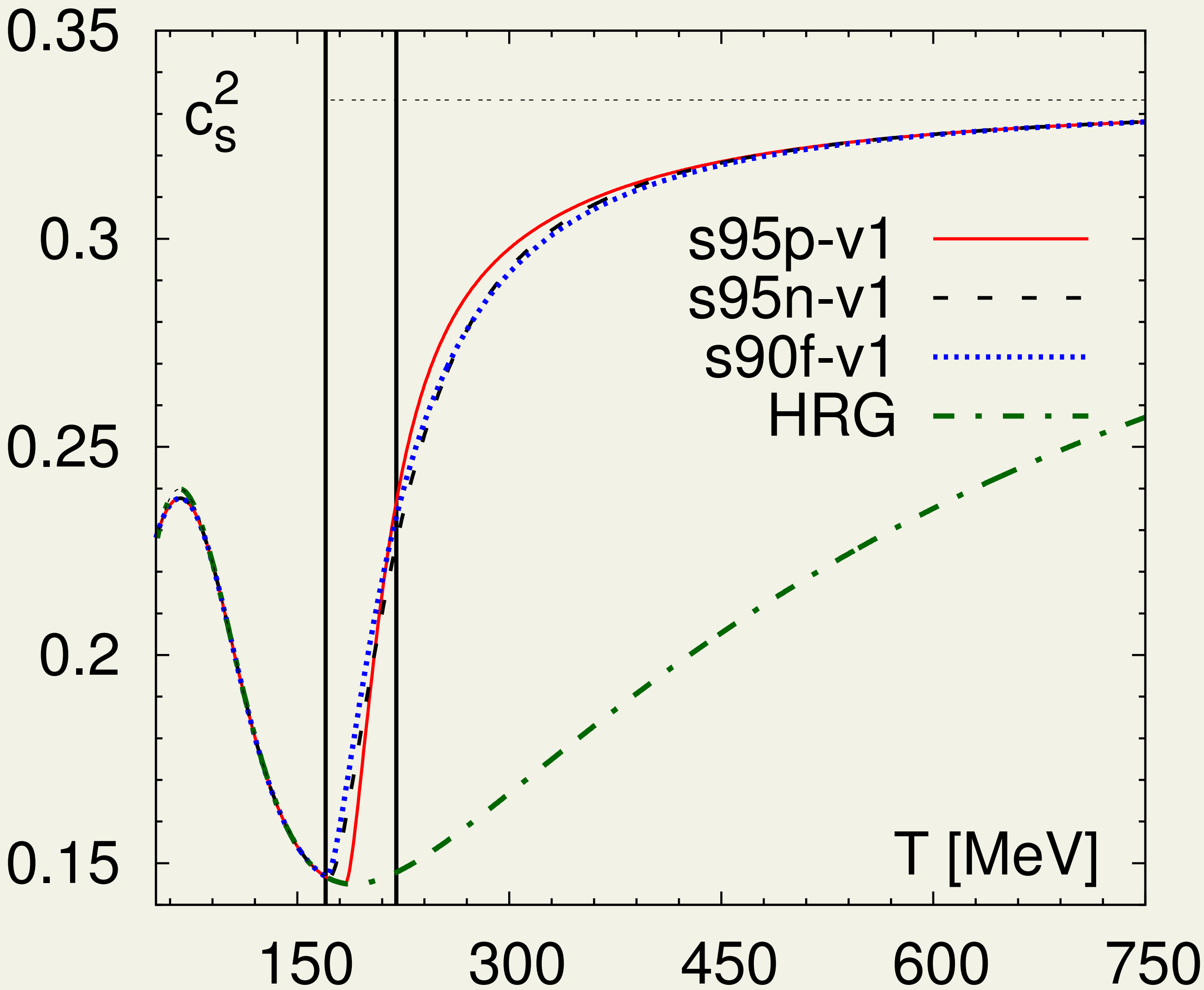
$$\frac{P}{T^4} - \frac{P_0}{T_0^4} = \int_{T_0}^{T} \mathrm{d}T' \, \frac{\epsilon - 3P}{T'^5}$$

# Interaction measure




- **For the 95%** $s_{\rm SB}$ **limit we get**
  $T_0 = 171.8$ **MeV,** $d_2 = 0.2654$, $d_4 = 6.563 \cdot 10^{-3}$,
  $c_1 = -4.370 \cdot 10^{-5}$, $c_2 = 5.774 \cdot 10^{-6}$, $n_1 = 8$, $n_2 = 9$

# Speed of sound




- **no softening below the HRG!**

# Relaxation times for mixtures

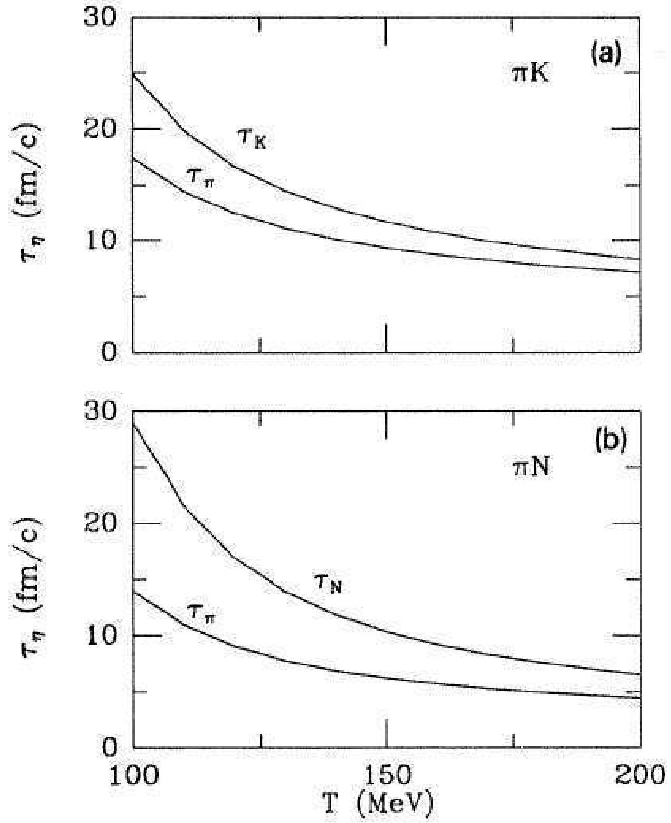



Fig. 13. Relaxation times of heat flow in (a) a $\pi$K mixture and (b) a $\pi$N mixture from eq. (5.4).

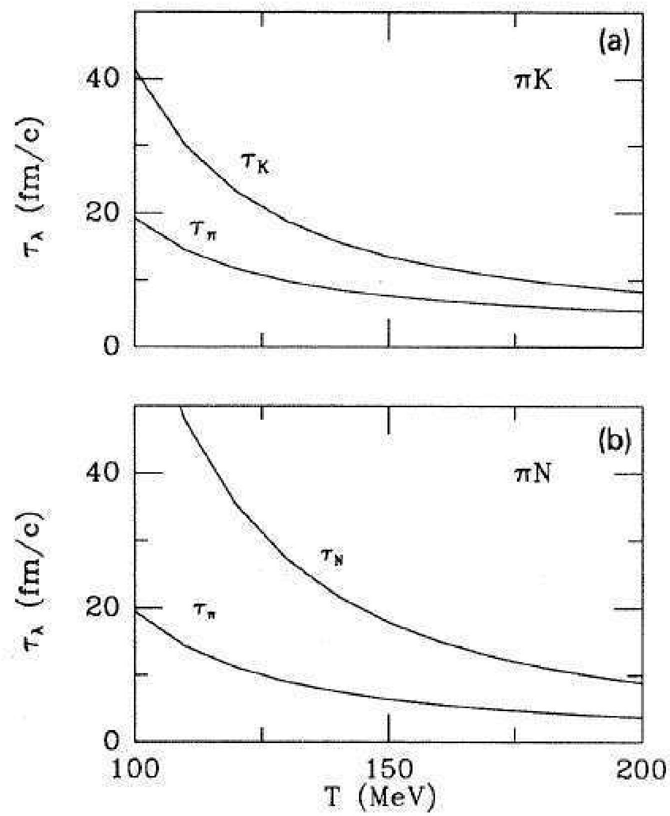
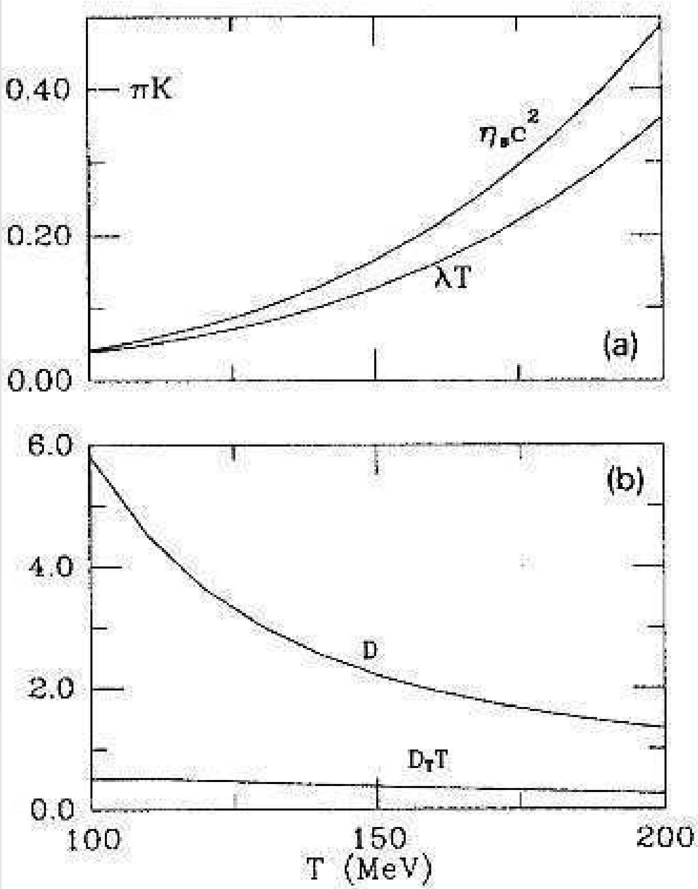



Fig. 14. Relaxation times of shear viscous flow in (a) a $\pi$K mixture and (b) a $\pi$N mixture from eq. (5.6).

Note: The ratio of nucleon to kaon relaxation times is nearly unity at $T = 200$ MeV, whereas at $T = 100$ MeV, the ratio is nearly $m_N/m_K$.

# Results for binary mixtures




. 10. Transport coefficients in a πK mixture using . (C.6) through eq. (C.9). (a) Shear viscosity and rmal conductivity. (b) Diffusion and thermal diffusion fficients.

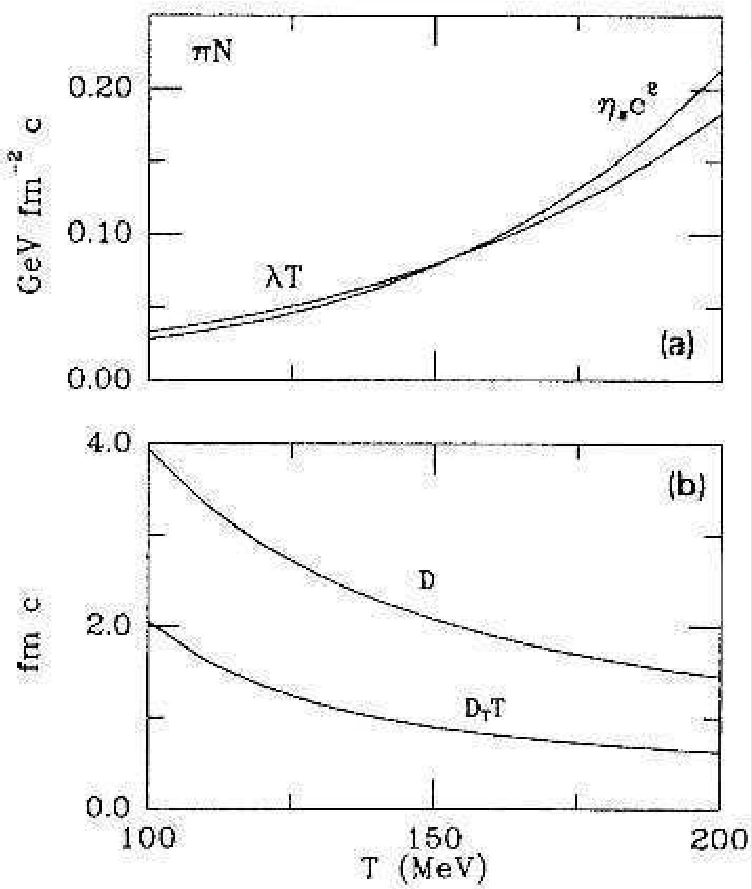



Fig. 11. Transport coefficients in a πN mixture. (a) Shear viscosity and thermal conductivity. (b) Diffusion and thermal diffusion coefficients.

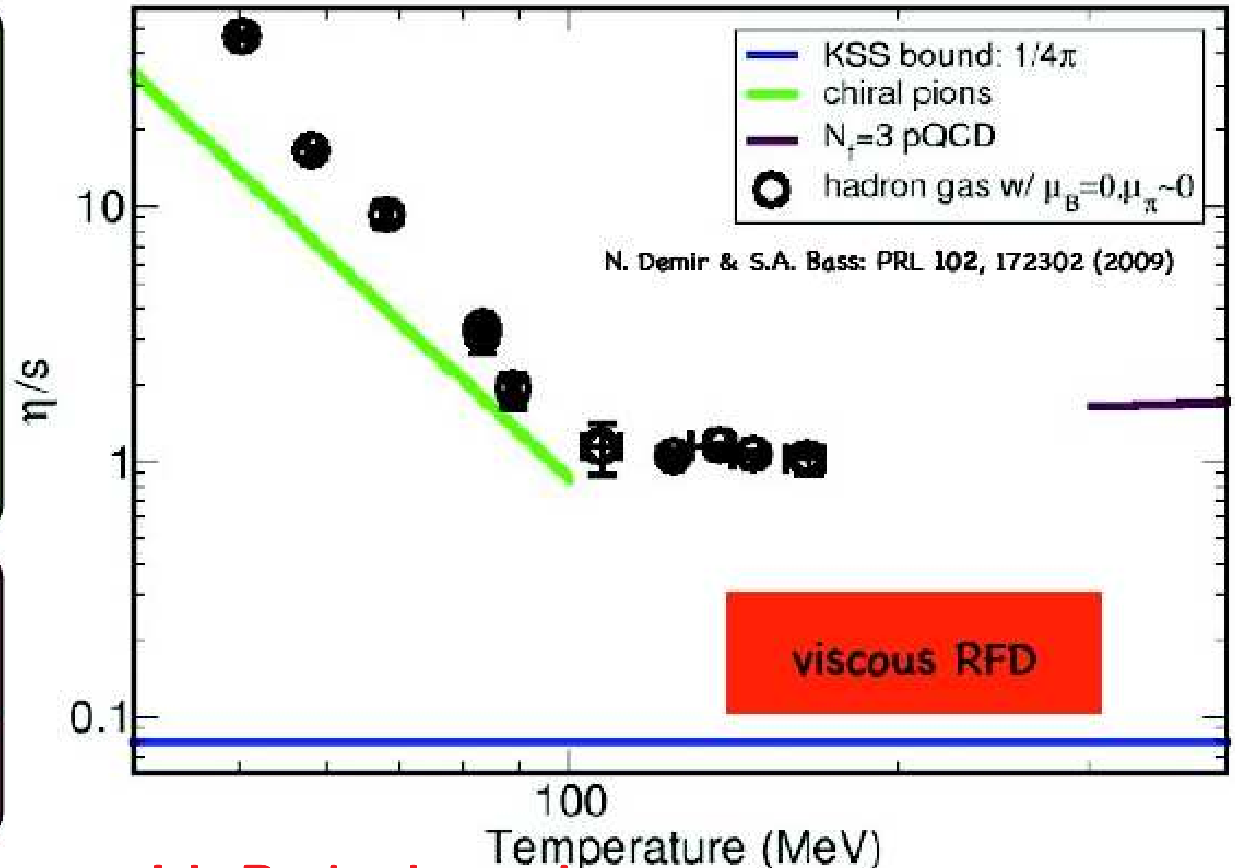
**Results for multicomponent systems, and inelastic interactions are coming.**

Bass

# η/s of a Hadron Gas

first reliable calculation of of η/s for a full hadron gas including baryons and anti-baryons

- low temperature trend qualitatively confirms chiral pion calculation
- above T=100 MeV: η/s≈1 remains roughly constant
- η/s is a factor of 3–5 above range required by viscous RFD analysis!




- breakdown of vRFD in the hadronic phase?
- what are the consequences for η/s in the deconfined phase?

**Would be nice to compare with Prakash et al…**

# Initial conditions

Venugopalan

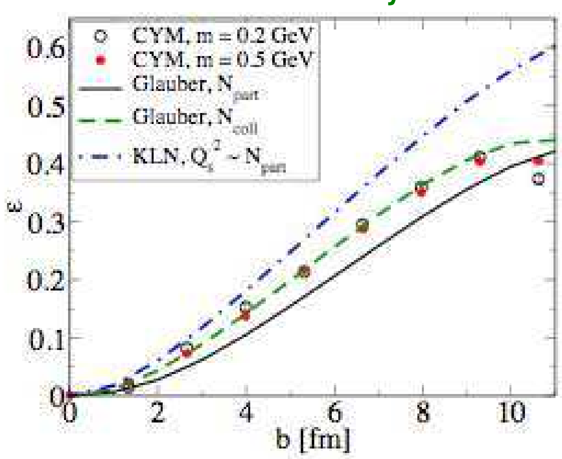
# Matching Glasma dynamics to Hydro

Classical field — Classical field / Particle — Particle

$$f \sim \frac{1}{\alpha_S} \qquad 1 \le f < \frac{1}{\alpha_S} \qquad f < 1 \qquad \tau$$

- Current matching of LO Glasma YM computations to hydro - "CGC initial conditions"- assumes instantaneous thermalization
- But $T_{\mu\nu}$ is far out of equilibrium in LO computations $p_z \sim 0$
- *No computations to date fully take into account NLO contributions that are as large as LO and should be resummed…*

eccentricity



Venugopalan

# Rapid isotropization in the Glasma

**Resum** $\left(\alpha_S \exp(\sqrt{Q_S \tau}\right)^n$ **- extend range of YM-dynamics**

Gelis,Lappi,RV (2008)

$$\langle T^{\mu\nu}(\tau,\eta,x_\perp)\rangle\Big|_{\substack{\text{LLog}\\ \text{resum}}} = \int [D\rho_1\rho_2]\, W_{Y_1}[\rho_1]\, W_{Y_2}[\rho_2] \times \int [Da]\, G_\tau[a]\, T^{\mu\nu}_{\text{LO}}[A_{\text{cl.}}+a]$$

"Holy Grail"
Spectrum of
small fluctuations

Fukushima,Gelis,McLerran (2006)

**Can also compute event by event initial conditions to estimate flow fluctuations:**

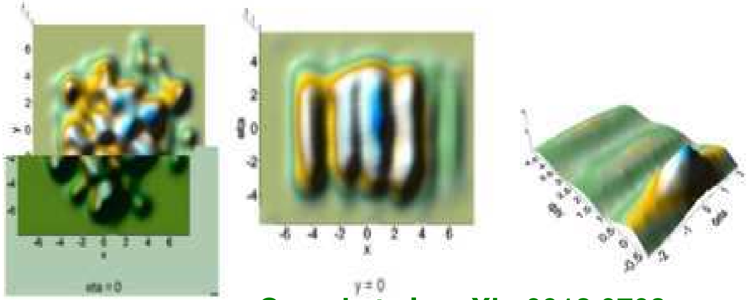
$$\left\langle T^{\mu_1\nu_1}(\tau_0,\eta_1,\vec{x}_{1\perp})\, T^{\mu_2\nu_2}(\tau_0,\eta_2,\vec{x}_{2\perp})\right\rangle$$

$$\left\langle T^{\mu_1\nu_1}(\tau_0,\eta_1,\vec{x}_{1\perp})\, T^{\mu_2\nu_2}(\tau_0,\eta_2,\vec{x}_{2\perp})\, T^{\mu_3\nu_3}(\tau_0,\eta_3,\vec{x}_{3\perp})\right\rangle$$

$\cdots$

**Similar in spirit to the event by event hydro code**

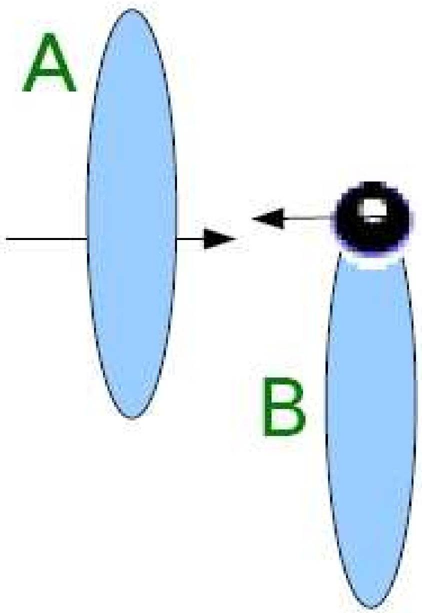
**NeXSPheRIO = NeXus + SPheRIO**



Grassi et al., arXiv:0912.0703

# fKLN

nucl-th/0605012
nucl-th/0611017



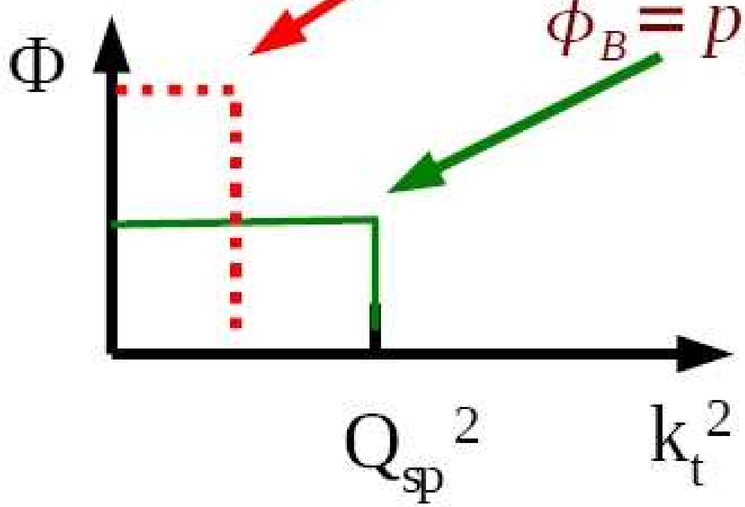

$$Q^2_{sB} = Q^2_{sp} T_B$$

$$T_B = \frac{\sum_{i\geq 0} p_i t_i}{\sum_{i\geq 1} p_i} = \frac{\langle T_B \rangle}{p_B}$$

$$\langle T_B \rangle (\vec{r}_\perp) = \int dz\, \rho_{WS}(z, \vec{r}_\perp)$$

$\phi_B = \phi(Q^2_{sp} \langle T_B(r_t) \rangle)$ ,

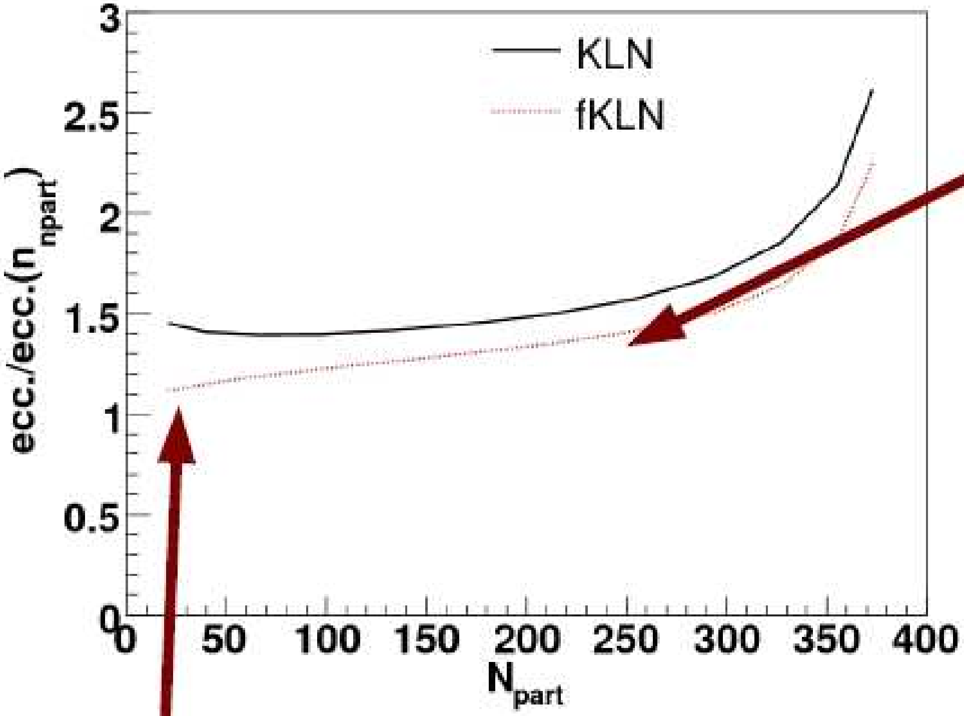
$\phi_B = p_B\, \phi(Q^2_{sp})$



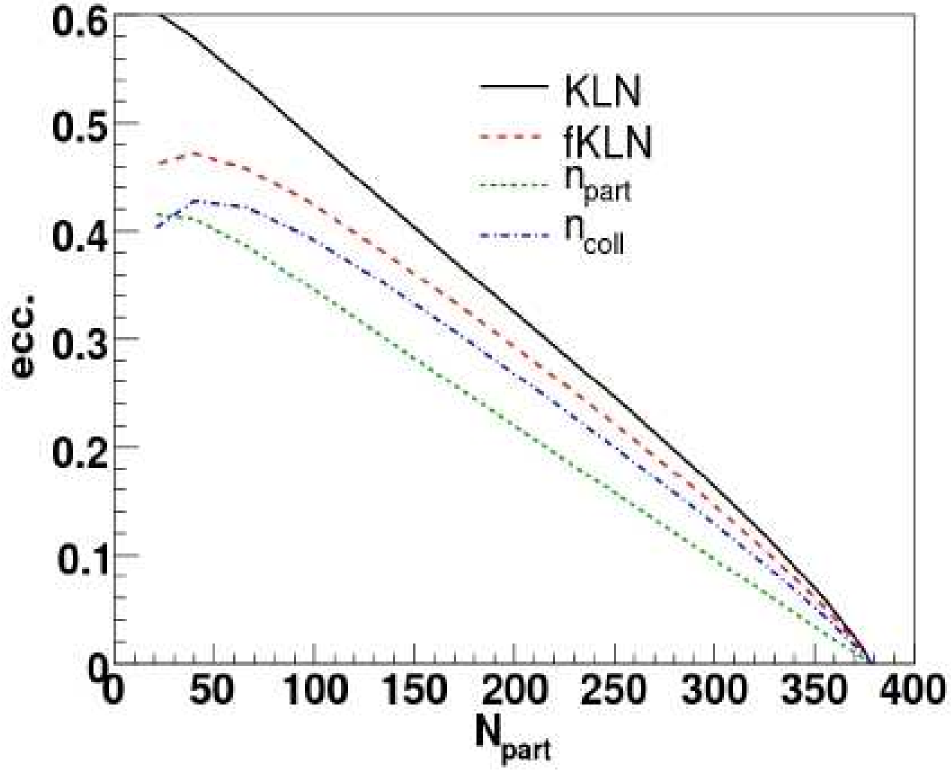

$$\phi_B(\langle T_B \rangle) \rightarrow p_B\, \phi_B(\langle T_B \rangle / p_B)$$

$$\frac{dN}{d^2 r_t} \sim p_A\, \phi_A \otimes p_B\, \phi_B$$

**Dumitru** **treatment of edge changes eccentricity**



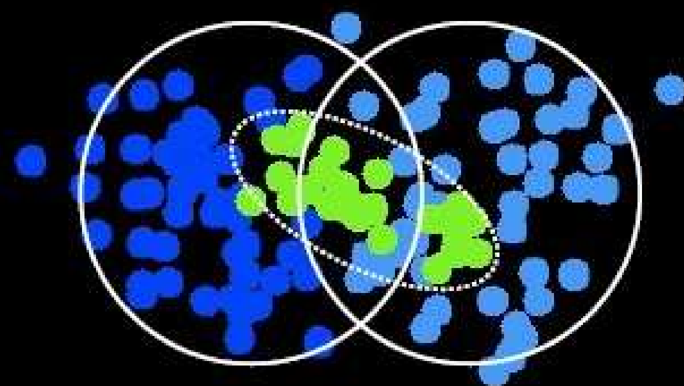

~ 30% effect, comparable to dissip. correction




fKLN approaches Glauber in peripheral collisions !

# Cu+Cu

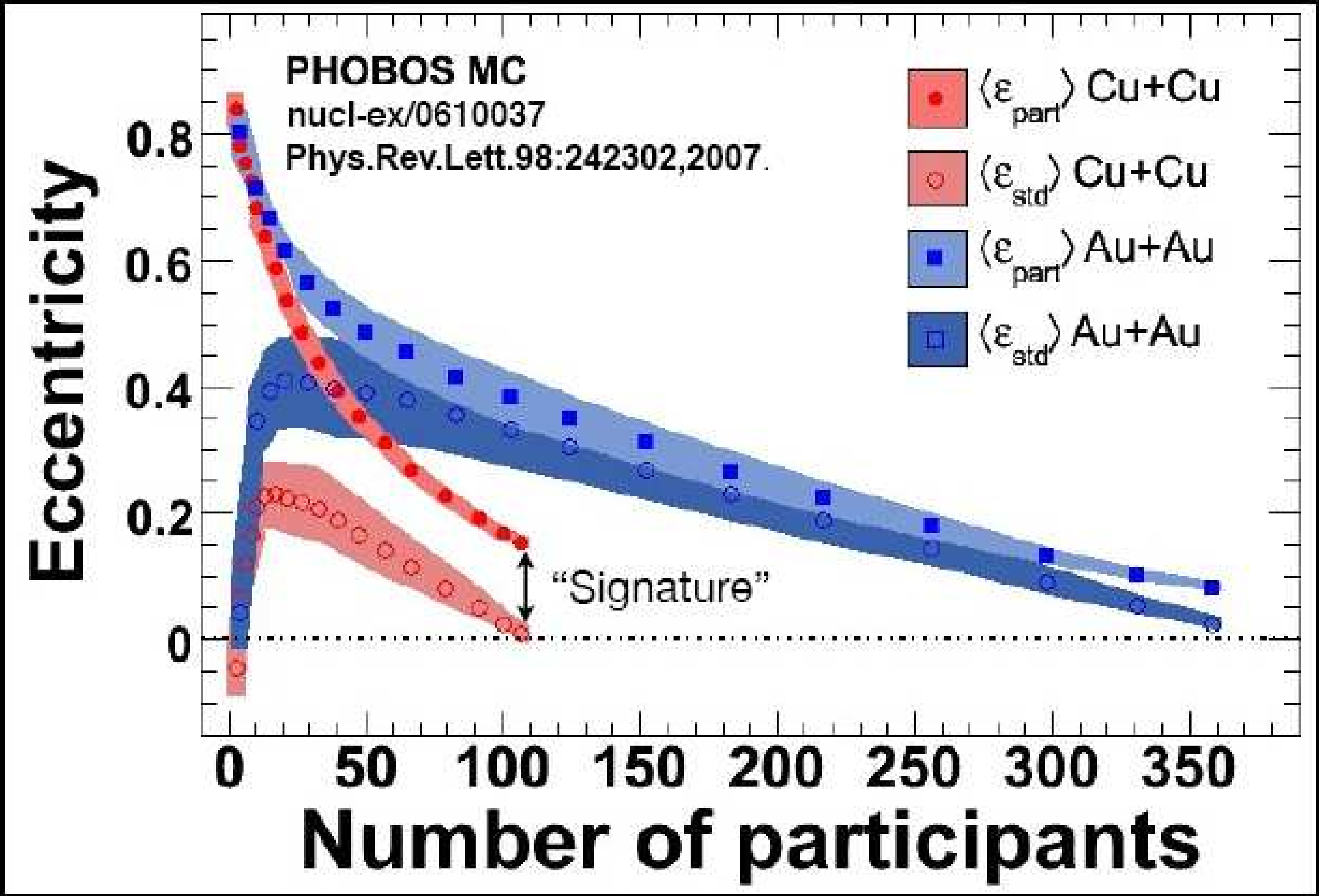
Principal axes make sense if $v_2$ depends on shape of produced matter (in SLP), not the reaction plane



$$\epsilon_{part} = \frac{\sigma_y'^2 - \sigma_x'^2}{\sigma_y'^2 + \sigma_x'^2} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4(\sigma_{xy}^2)^2}}{\sigma_y^2 + \sigma_x^2}$$

"Participant eccentricity"

# Participant vs. Standard




If you see ε~0 in central collisions, then you are using the wrong eccentricity, or not including fluctuations

# Public Versioning is Fundamental

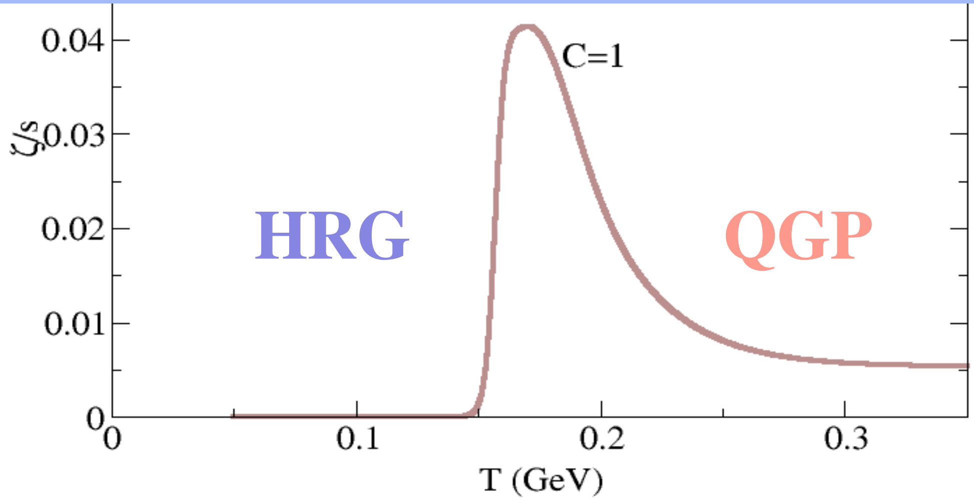
- **MC-KLN is an example where clear versioning should be used on figures, but it applies to all initial state calculations**
- **Model: we always refer to PDFs by their full names in publications**
  - MRST2004, CTEQ6.1M, EKS98, EPS08, etc.
- **Our models of the initial state should be treated in the same way**
  - TGlauberMC v1.2
  - MC-KLN vs. 1.01
- **Then you are not just using "CGC" initial conditions, but a particular implementation**
  - Will allow theory-to-theory comparisons as well as theory-to-experiment

# Viscous hydrodynamics and bulk viscosity

Song

# bulk viscosity and relaxation time

Bulk viscosity:




Relaxation times: $\tau_\Pi \sim \zeta$ also peaks near $T_c$,

**this plays an important role for bulk viscous dynamics**

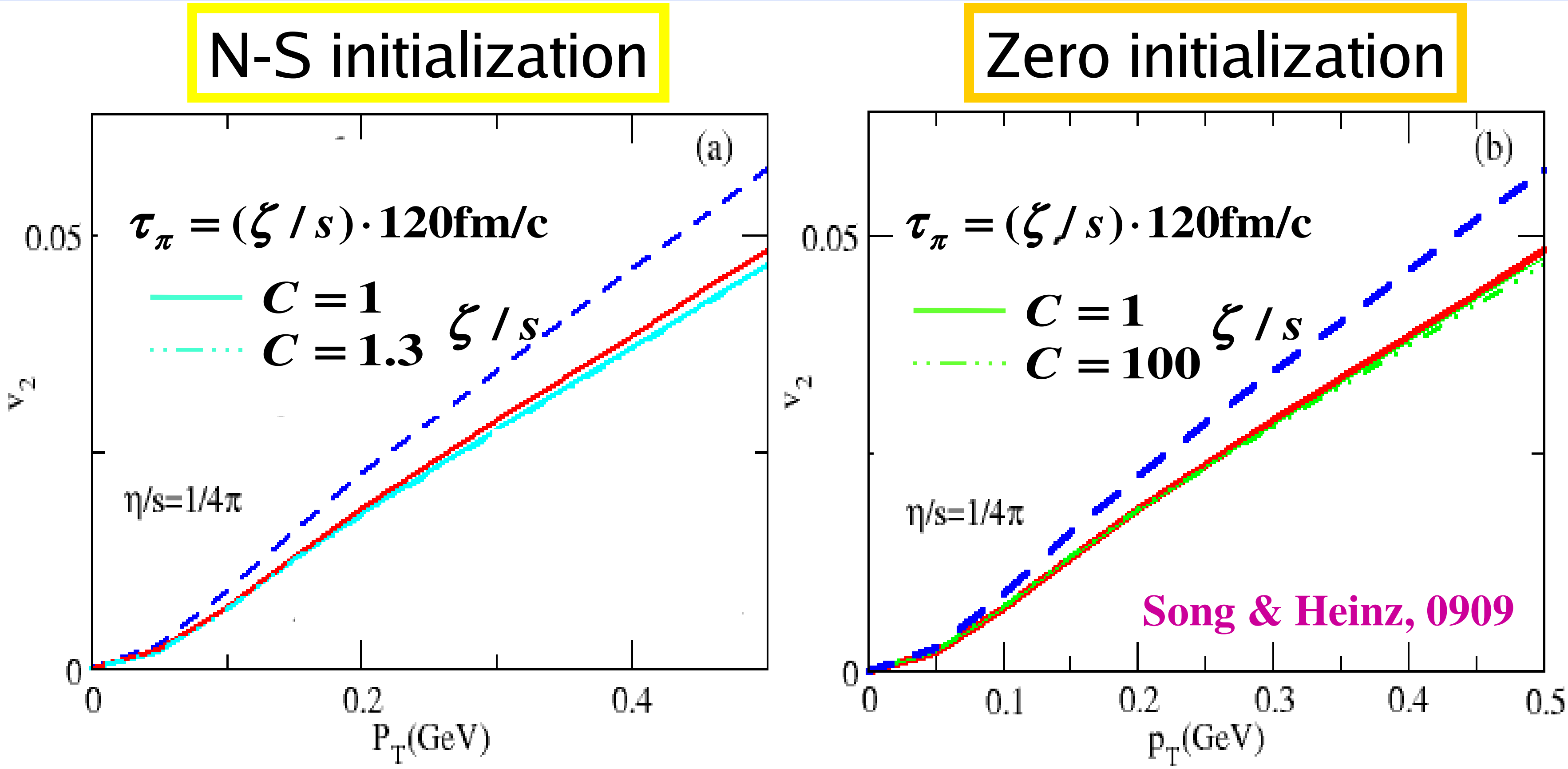
**N-S initialization:** $\Pi_0 = -\zeta(\partial \cdot u)$

large $\tau_\Pi$ near $T_c$ $\longrightarrow$ **keeps large negative value of** $\Pi$ in phase transition region

$\longrightarrow$ **viscous hydro breaks down** $(p + \Pi < 0)$ for larger $\zeta / s$

**viscous hydro is only valid with small** $\zeta / s$ $\longrightarrow$ **small bulk viscous effects on $v_2$**

# Uncertainties from **bulk viscosity**



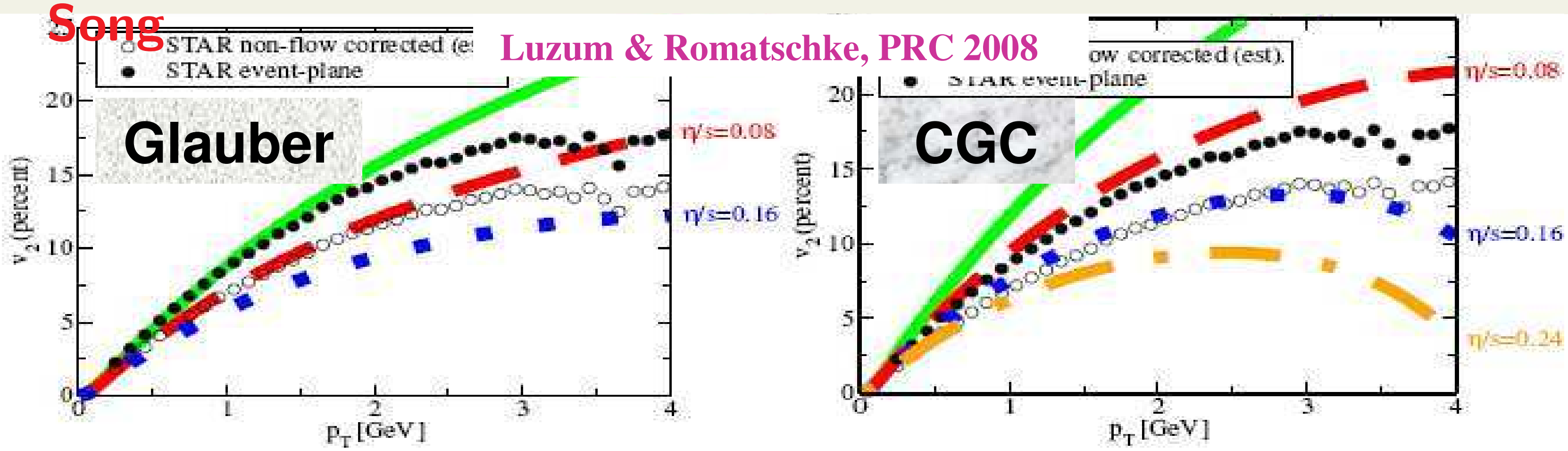

-with a critical slowing down $\tau_\Pi$, effects from bulk viscosity effects are much smaller than from shear viscosity

bulk viscosity influences $v_2$ ~5% (N-S initial.) <4% (zero initial.)

⟺ uncertainties to $\eta / s$ ~20% (N-S initial.) <15% (zero initial.)

Song

Luzum & Romatschke, PRC 2008




(uncertainties in $\eta/s$ )

**-initial conditions:** CGC vs. Glauber **~100%** **recent progress: Heinz et al. 0907**

**-EoS:** EOS Q, vs. EOS L **~25%**

**-chemical composition of HRG :** (PEC vs. CE) **~100%**

**-viscosity of HRG** (or equil. HRG vs. non-equil. HRG) **: ~100-150%**

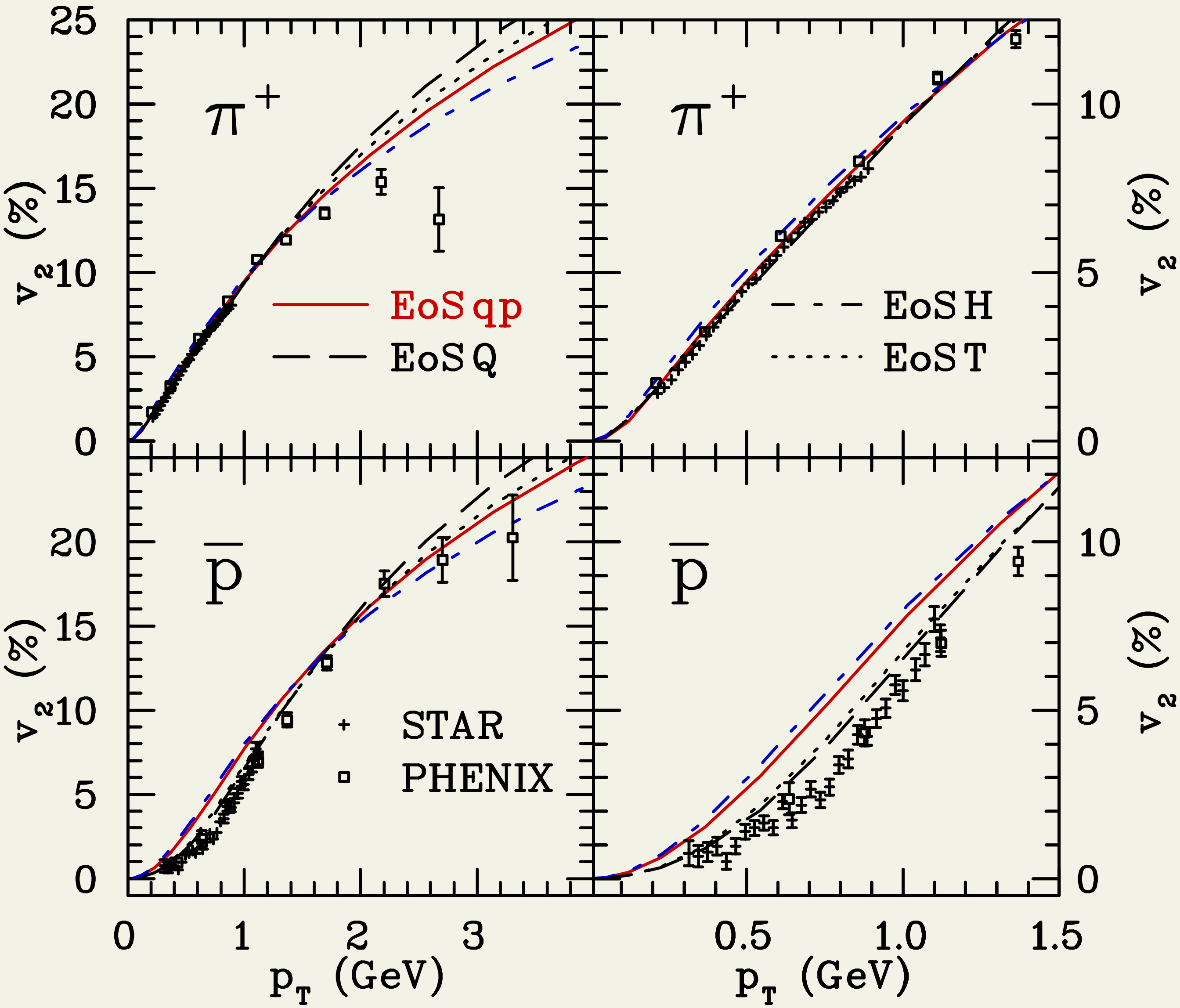
**-bulk viscosity: ~20%**

**conservative upper limit:** $\eta/s \leq 5\times(1/4\pi)$

-To further decrease the uncertainties from bulk viscosity, (or to extract both shear & bulk viscosity from exp. data), **one need more sensitive exp. observables**

# May be worse, I think

**Little difference between lattice EOS parameterization and a hadron gas!**




Huovinen, NPA761, 296 ('05)

**Q: bag model**

**qp: lattice fit ($T_c = 170$ MeV)**

**H: hadron gas**

**T: interpolated $\varepsilon(T)$ between hadron gas and $\varepsilon \propto T^4$ plasma**

# Viscous freezeout - “Delta $f$”

# Introduction

- How does viscosity affects observables?

One needs a convertor of flow field into particles at freezeout

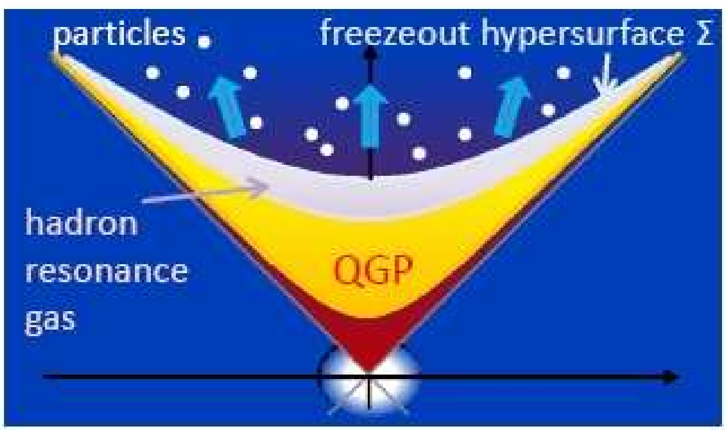
hydro result　　observables

Cooper-Frye formula

$$\boxed{\frac{d^2 N_i}{d^2 p_T dy} = \frac{g_i}{(2\pi)^3} \int_\Sigma p_i^\mu d\sigma_\mu (f_0^i + \delta f^i)}$$

variation of the flow/hypersurface

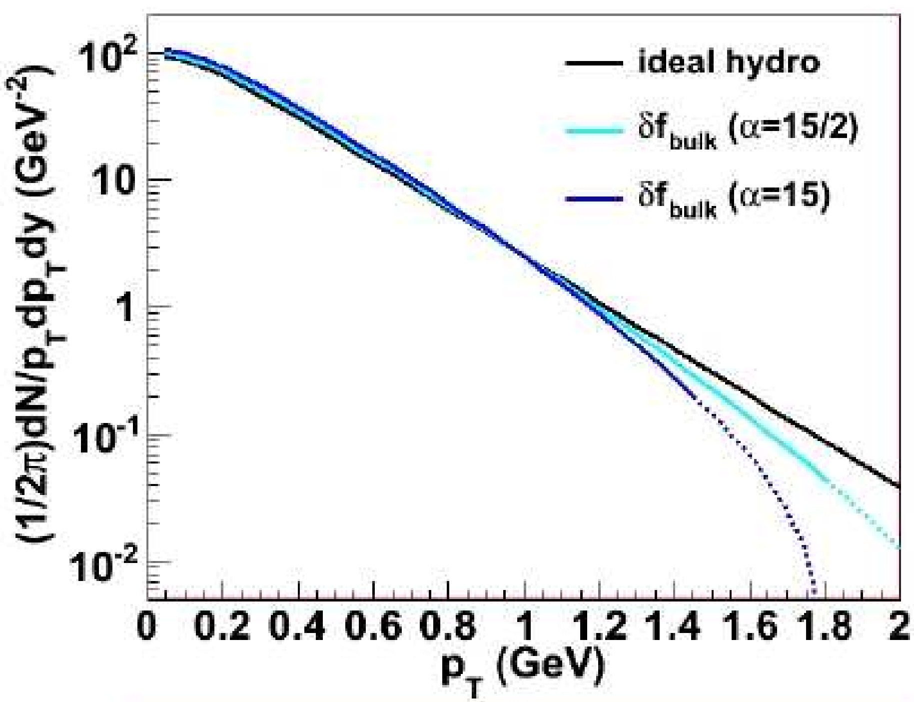
modification of the distribution

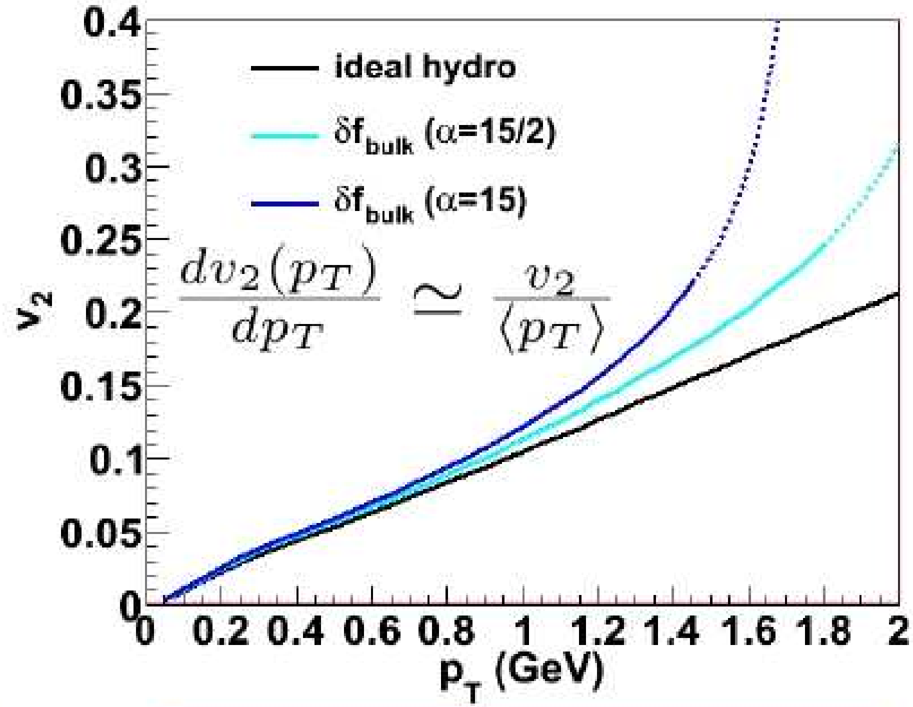



We need to estimate both $\delta f^i$ and $\delta u^\mu$ in a multi-component system

# Bulk Viscosity and Particle Spectra

- *Au+Au*, $\sqrt{s_{NN}} = 200(\mathrm{GeV})$, *b* = 7.2(fm), $p_T$-spectra and $v_2(p_T)$ of $\pi^-$




$$\delta f^i = -f_0^i(1 \pm f_0^i)[p_i^\mu \varepsilon_\mu + p_i^\mu p_i^\nu \varepsilon_{\mu\nu}]$$

$v_2(p_T)$ ⇨ enhanced

Even "small" bulk viscosity may have significant effects on particle spectra

# Solution

$$\boxed{f_{(i)}(p) = f_{0(i)}\left[1+(1\mp f_{0(i)})\left(\varepsilon_{(i)} + \varepsilon_{\mu(i)} p^{\mu}_{(i)} + \varepsilon_{\mu\nu(i)} p^{\mu}_{(i)} p^{\nu}_{(i)}\right)\right]}$$

$$\varepsilon_{(i)} = E_{0(i)} \Pi_{(i)}$$

$$\varepsilon^{\mu}_{(i)} = D_{0(i)} \Pi_{(i)} u^{\mu} + D_{1(i)} q^{\mu}_{(i)} + D_{2(i)} \nu^{\mu}_{(i)}$$

$$\varepsilon^{\mu\nu}_{(i)} = B_{0(i)} \left(\Delta^{\mu\nu} - 3u^{\mu}u^{\nu}\right)\Pi_{(i)} + 2B_{1(i)} u^{(\mu} q^{\nu)}_{(i)} + 2B_{3(i)} u^{(\mu} \nu^{\nu)}_{(i)} + B_{2(i)} \pi^{\mu\nu}_{(i)}$$

$$\boxed{B_{2(i)} = \frac{1}{2J_{42(i)}}} \longrightarrow$$ shear viscosity

Boltzmann Gas

$$\boxed{B_{2(i)} = \frac{1}{2\left(\varepsilon_{(i)} + P_{(i)}\right)T^2}}$$

$$J_{nq(i)} = \frac{1}{(2q-1)!!} \int d\omega_{(i)} E^{n-2q}_{(i)} K^{2q} f_{0(i)} \left(1 \quad a f_{0(i)}\right)$$

Denicol

# Closed Equations – Bulk and Shear

$$
\begin{aligned}
\frac{d\Pi_{(i)}}{d\tau} + \frac{\Pi_{(i)}}{\tau_{\Pi(i)}} + \sum_{j\neq i} \frac{\Pi_{(j)}}{\tau_{\Pi(i)(j)}} = & -\left(\beta_{\zeta(i)} + \zeta_{\Pi\Pi(i)}\Pi_{(i)}\right)\theta + \zeta_{\Pi\pi(i)}\pi^{\mu\nu}_{(i)}\sigma_{\mu\nu} \\
& -\zeta_{\Pi n(i)}\partial_\mu n^\mu - \alpha_{\Pi n(i)} n^\mu_{(i)}\dot{u}_\mu - \beta_{\Pi n(i)} n^\mu_{(i)}\nabla_\mu \alpha_{0(i)} \\
& -\zeta_{\Pi q(i)}\partial_\mu q^\mu - \alpha_{\Pi q(i)} q^\mu_{(i)}\dot{u}_\mu - \beta_{\Pi q(i)} q^\mu_{(i)}\nabla_\mu \alpha_{0(i)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\pi^{\langle\mu\nu\rangle}_{(i)}}{d\tau} + \frac{\pi^{\mu\nu}_{(i)}}{\tau_{\pi(i)}} + \sum_{j\neq i}\frac{\pi^{\mu\nu}_{(j)}}{\tau_{\pi(i)(j)}} = & \; 2\left(\beta_{\eta(i)} + \eta_{\pi\Pi(i)}\Pi_{(i)}\right)\sigma^{\mu\nu} - 2\eta_{\pi\pi(i)}\pi^{\langle\mu}_{\alpha(i)}\sigma^{\nu\rangle\alpha} + 2\pi^{\langle\mu}_{\alpha(i)}\omega^{\nu\rangle\alpha} - \left(\frac{1}{2} + \frac{7}{6}\eta_{\pi\pi(i)}\right)\pi^{\mu\nu}_{(i)}\theta \\
& + 2\eta_{\pi n(2)(i)}\nabla^{\langle\mu} n^{\nu\rangle}_{(i)} + 2\beta_{\pi n(i)} n^{\langle\mu}_{(i)}\nabla^{\nu\rangle}\alpha_{0(i)} - 2\alpha_{\pi n(i)} n^{\langle\mu}_{(i)}\dot{u}^{\nu\rangle} \\
& + 2\eta_{\pi q(2)(i)}\nabla^{\langle\mu} q^{\nu\rangle}_{(i)} + 2\beta_{\pi q(i)} q^{\langle\mu}_{(i)}\nabla^{\nu\rangle}\alpha_{0(i)} - 2\alpha_{\pi q(i)} q^{\langle\mu}_{(i)}\dot{u}^{\nu\rangle}
\end{aligned}
$$

Eqs. will depend on

$\Pi_{(i)}, q^\mu_{(i)}, \pi^{\mu\nu}_{(i)}, \nu^\mu_{(i)}$ **MORE variables!**

$\Pi, q^\mu, \pi^{\mu\nu}, \nu^\mu_r$ (possible?)

**in general - for one-component massless gas, with viscous shear only**

$$\delta f \equiv f_{eq} \times C(\chi)\, \pi^{\mu\nu} \frac{p_\mu p_\nu}{T^2}\, \chi(\frac{p}{T})$$

**from Grad's ansatz:** $\chi \equiv 1$

**this was a starting point in deriving IS hydro from kinetic theory**

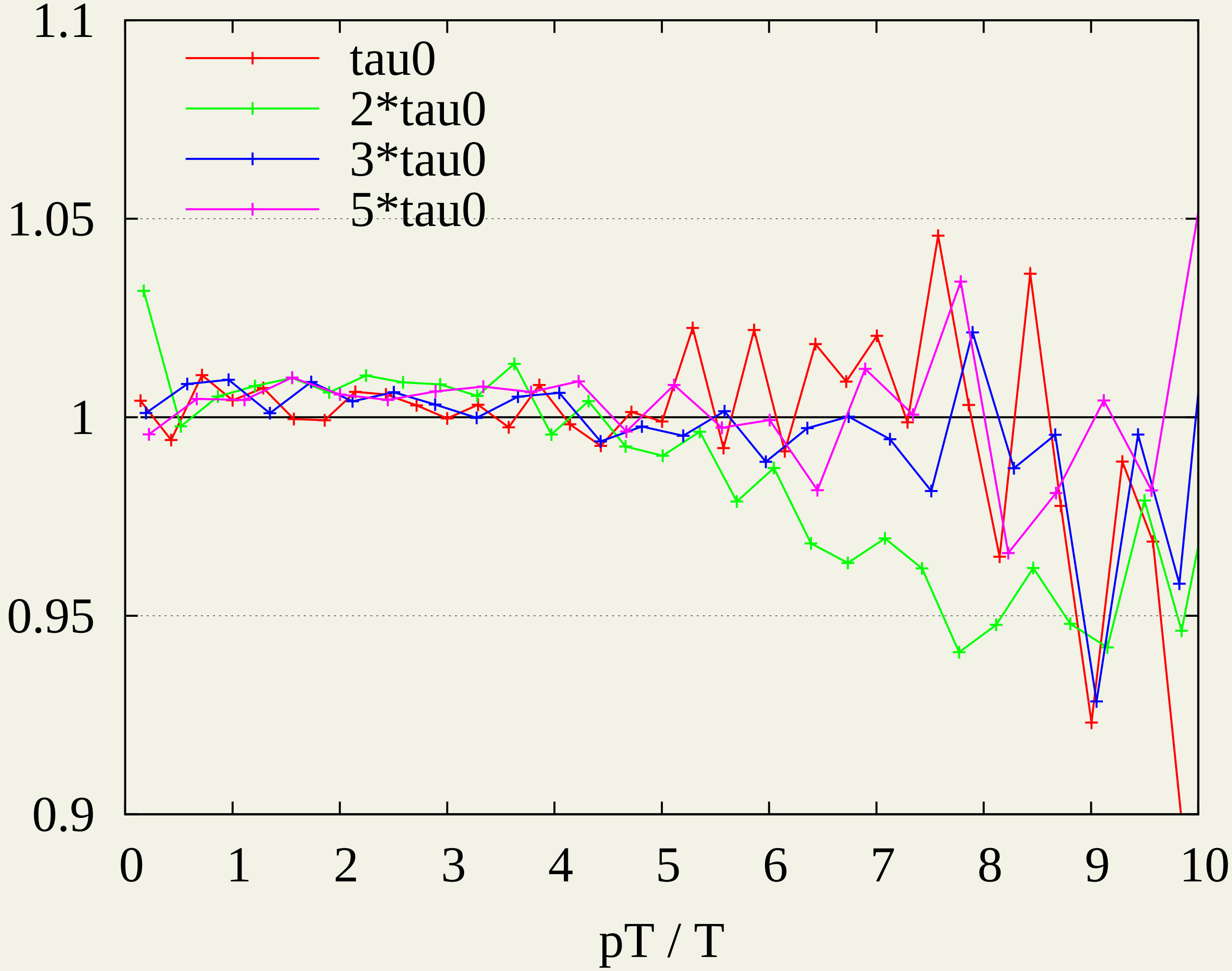
**from linear response:** $\chi(x) \sim x^\alpha$ **with** $-1 \lesssim \alpha \lesssim 0$ Dusling, Teaney, Moore, ('09)

$\delta f$ **blows up at large momenta** $\Rightarrow$ **approximation breaks down**

**transport can tell how far in momenta we can trust these...**

**ratio - transport spectra / Grad approximation** $\eta/s \sim 0.08$

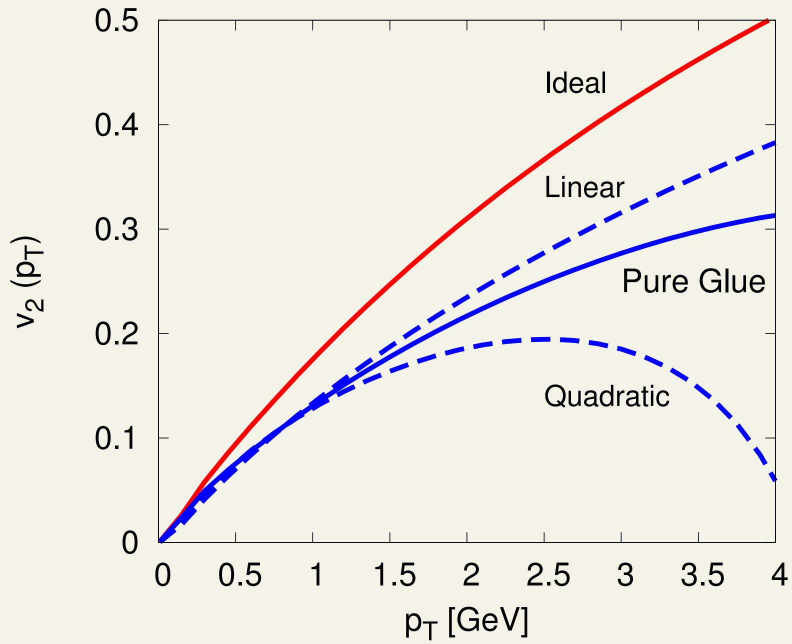
DM ('09):




**spectra are few-percent accurate even at** $p_T/T = 6$**(!)**

## Phenomenological Summary

pure glue, $\eta/s = 0.08$, $e_{\rm frz} = 0.6\,{\rm GeV/fm^3}$

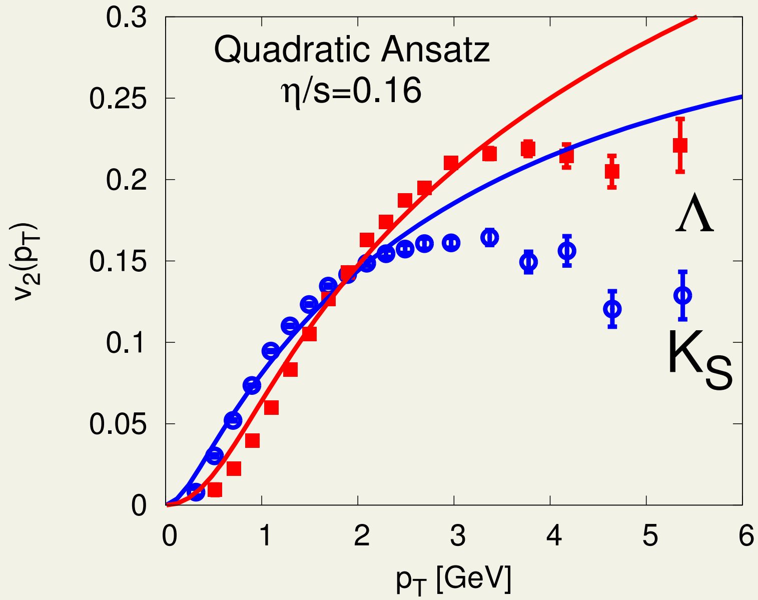


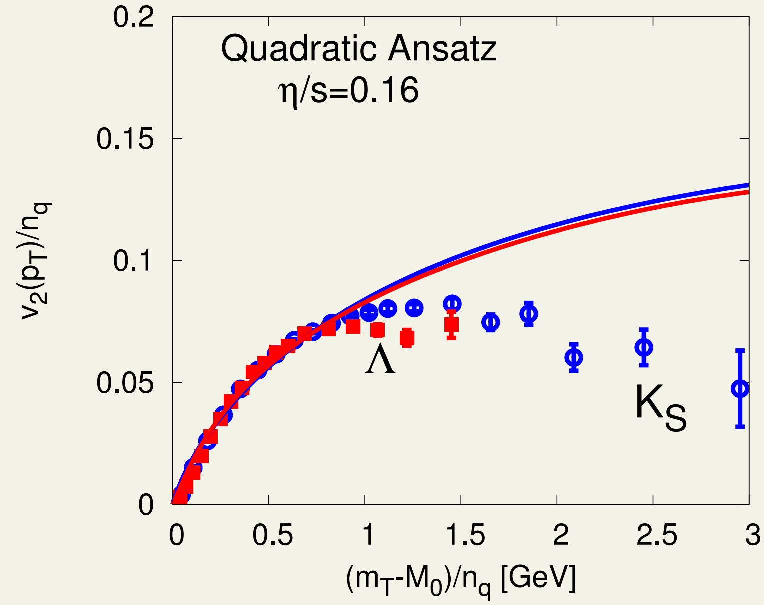

pQCD is closer to a linear ($\tau_R =$ const) rather than a quadratic ansatz

**how high in pT can one trust this? (no jets)**

**Teaney** $\sigma_B/\sigma_M \approx 1.5$ **works - just like additive quark model for** Bleicher et al..

Scaling



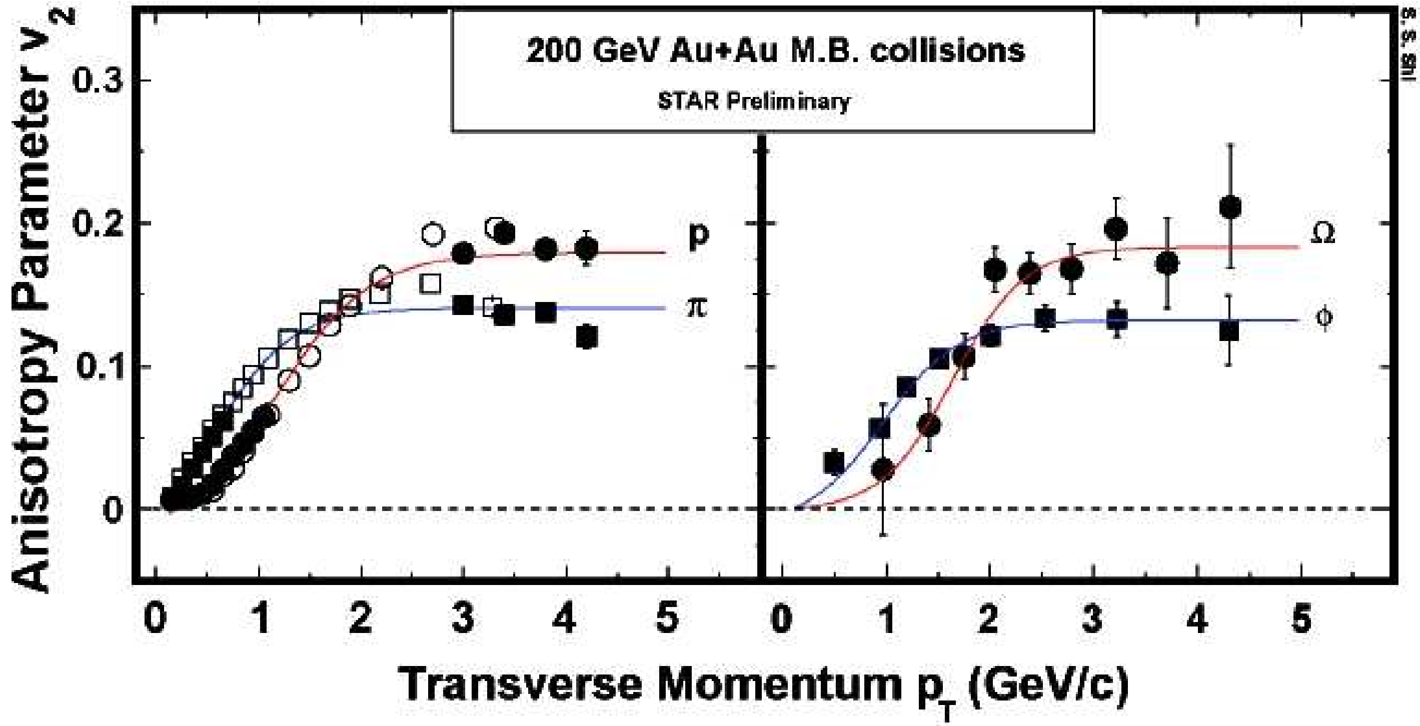




Perhaps quark number scaling is simply *Relaxation Time Scaling* (RTS)

$$\tau_{M,B} \propto 1/\sigma_{M,B}$$

Teaney

# Partonic Collectivity



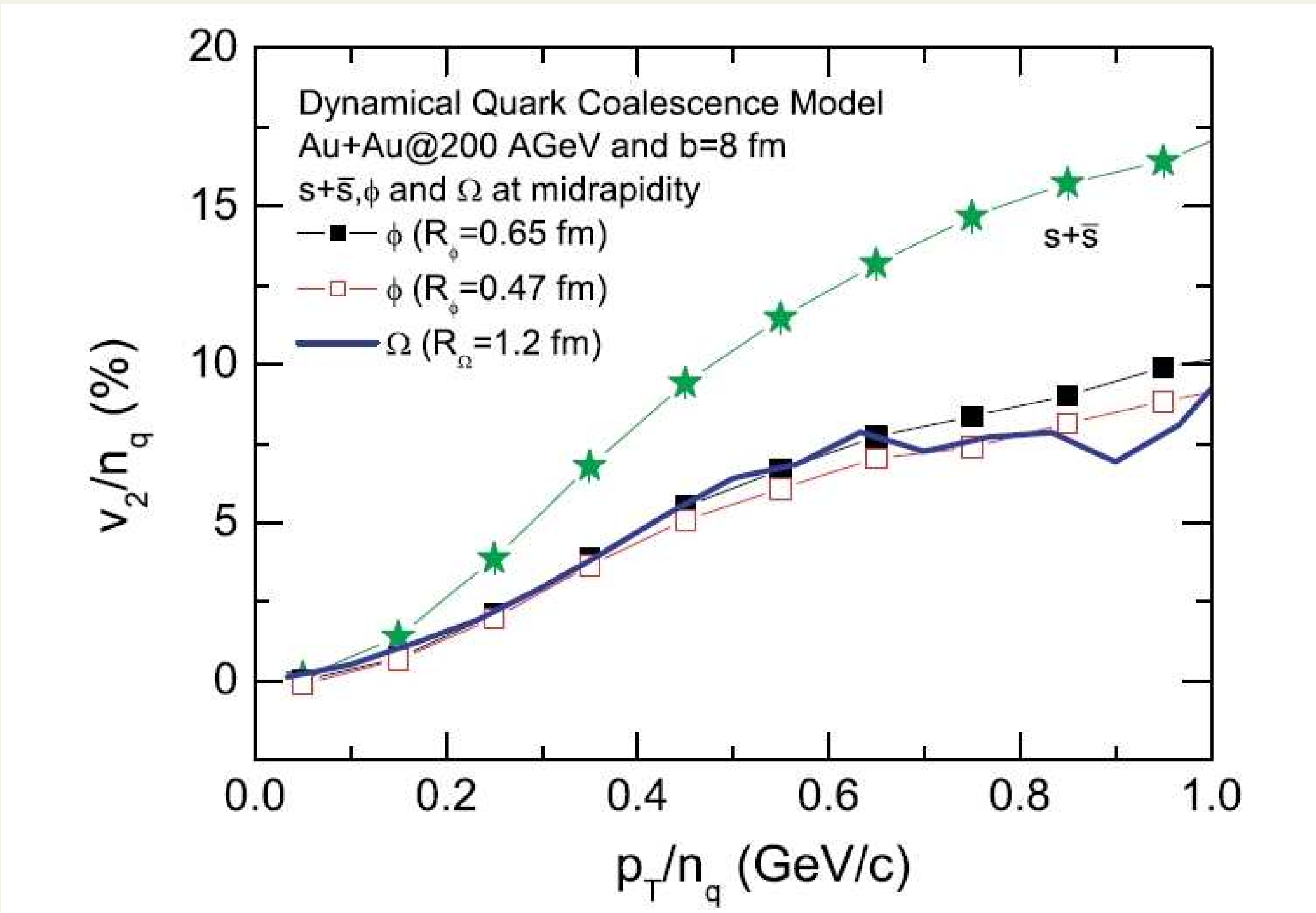

PHENIX π and p: nucl-ex/0604011v1
NQ inspired fit: X. Dong et al. Phy. Let. B 597 (2004) 328-332

**Partonic collectivity at RHIC : case closed.**

## closed - really?

**simple scaling formulas do not follow from a dynamical coalescence approach**

Chen & Ko, PRC73 ('06)



## Viscous hydro revisions

a rather heuristic "derivation"

# Third order hydrodynamics

$$\boxed{s^\mu = s_0 u^\mu - \beta_2 \pi_{\mu\nu} \pi^{\mu\nu} \frac{u^\mu}{2T} + \alpha \beta_2^2 \pi_{\alpha\beta} \pi^\alpha_\sigma \pi^{\beta\sigma} \frac{u^\mu}{T}}$$

$$T \partial_\mu s^\mu = \overbrace{\pi_{\mu\nu} \left( \sigma^{\mu\nu} - \beta_2 \dot{\pi}^{\mu\nu} - \frac{1}{2} T \partial_\alpha \left( \frac{\beta_2}{T} u^\alpha \right) \pi^{\mu\nu} \right)}^{\textit{Israel-Stewart}} +$$

$$+ \alpha T \pi_{\alpha\beta} \pi^\alpha_\sigma \pi^{\beta\sigma} \partial_\mu \left( \beta_2^2 \frac{u^\mu}{T} \right) + 3 \alpha \beta_2^2 \pi_{\alpha\beta} \pi^\alpha_\sigma \dot{\pi}^{\beta\sigma}$$

with

$\sigma^{\mu\nu} = \nabla^{<\mu} u^{\nu>}$ and $\dot{\pi}^{\alpha\beta} = u^\mu \partial_\mu \pi^{\alpha\beta}$

Enforcing the second law of thermodynamics $\partial_\mu s^\mu \geqslant 0$ ,

one obtains a third-order evolution equation for $\pi^{\mu\nu}$

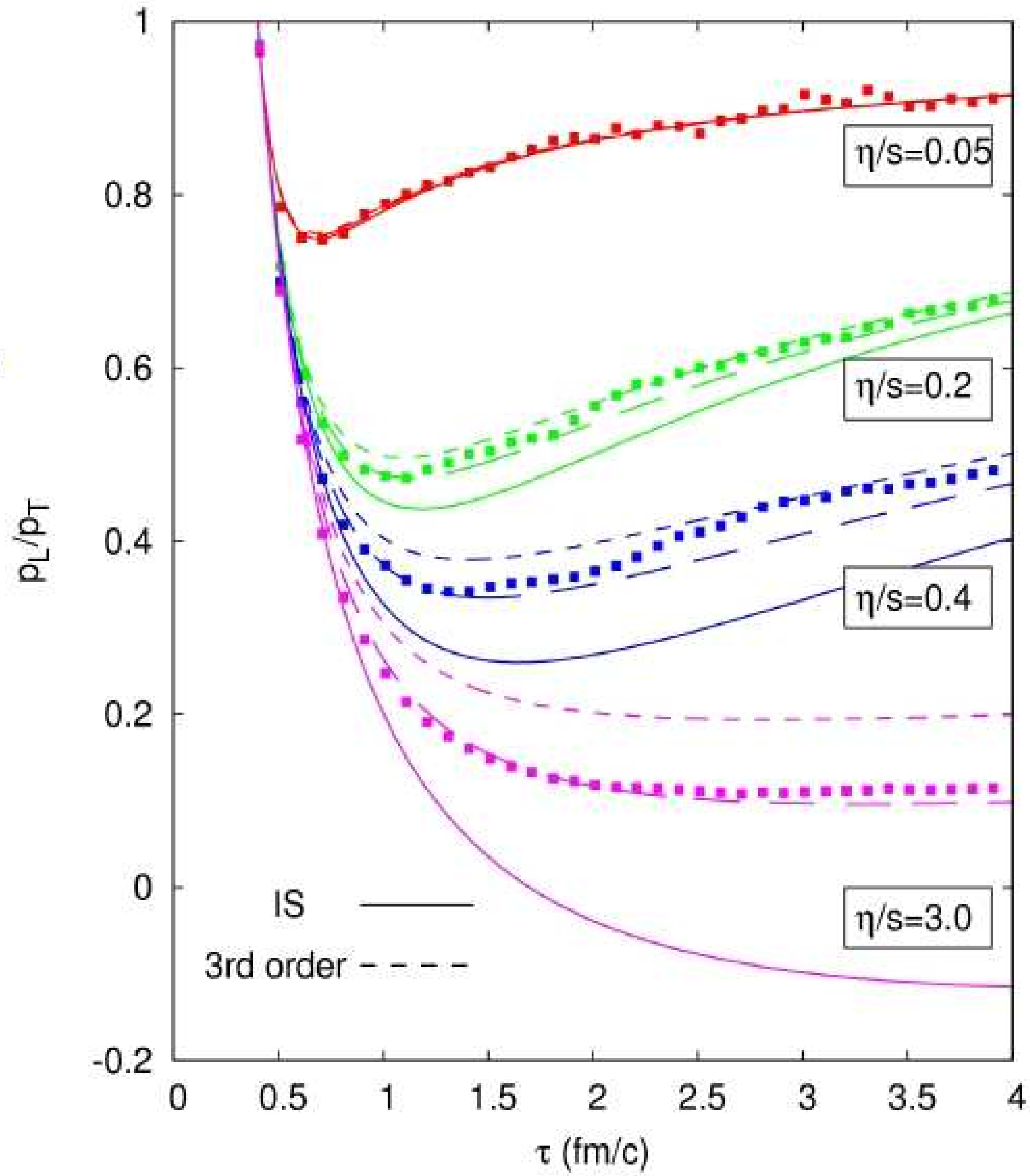
*( s. arXiv:0907.4500 for details )*

**El but looks promising - does it work in general?**

# Hydro vs BAMPS

$$\dot{\pi} = -\frac{\pi}{\tau_\pi} - \frac{4}{3}\frac{\pi}{\tau} + \frac{8}{27}\frac{e}{\tau} - 3\frac{\pi^2}{e\tau}$$

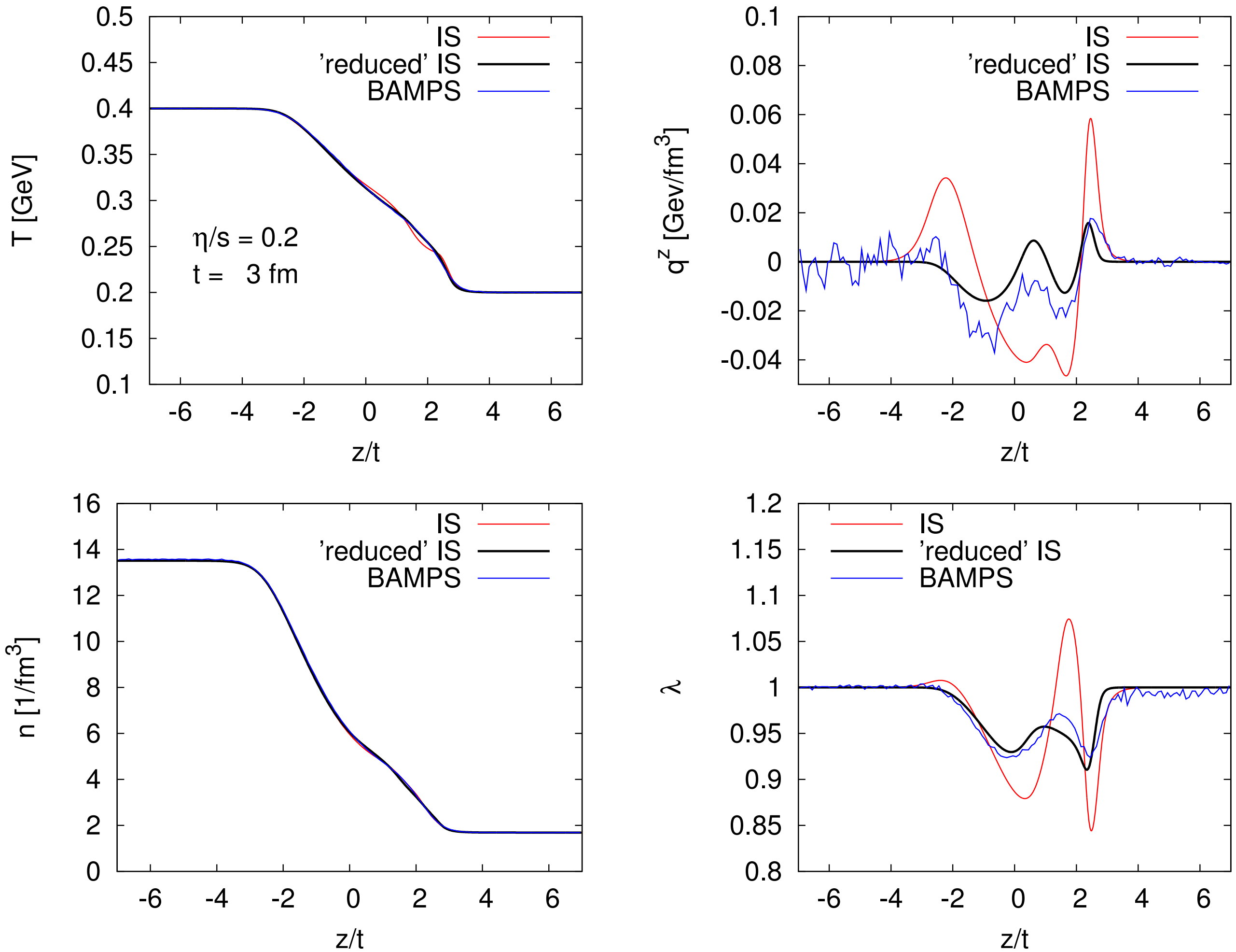
$$\dot{\pi} = -\frac{\pi}{\tau_\pi} - \frac{4}{3}\frac{\pi}{\tau} + \frac{8}{27}\frac{e}{\tau} - \frac{5}{3}\frac{\pi^2}{e\tau}$$




Including “higher-orders”:

*HO terms are taken by maximum value → good agreement at maximum dissipation*

## smoothed Riemann problem: Heat flow $\eta/s = 0.2$ $t = 3.0$ fm



# Niemi - maybe Israel-Stewart is not the right theory?

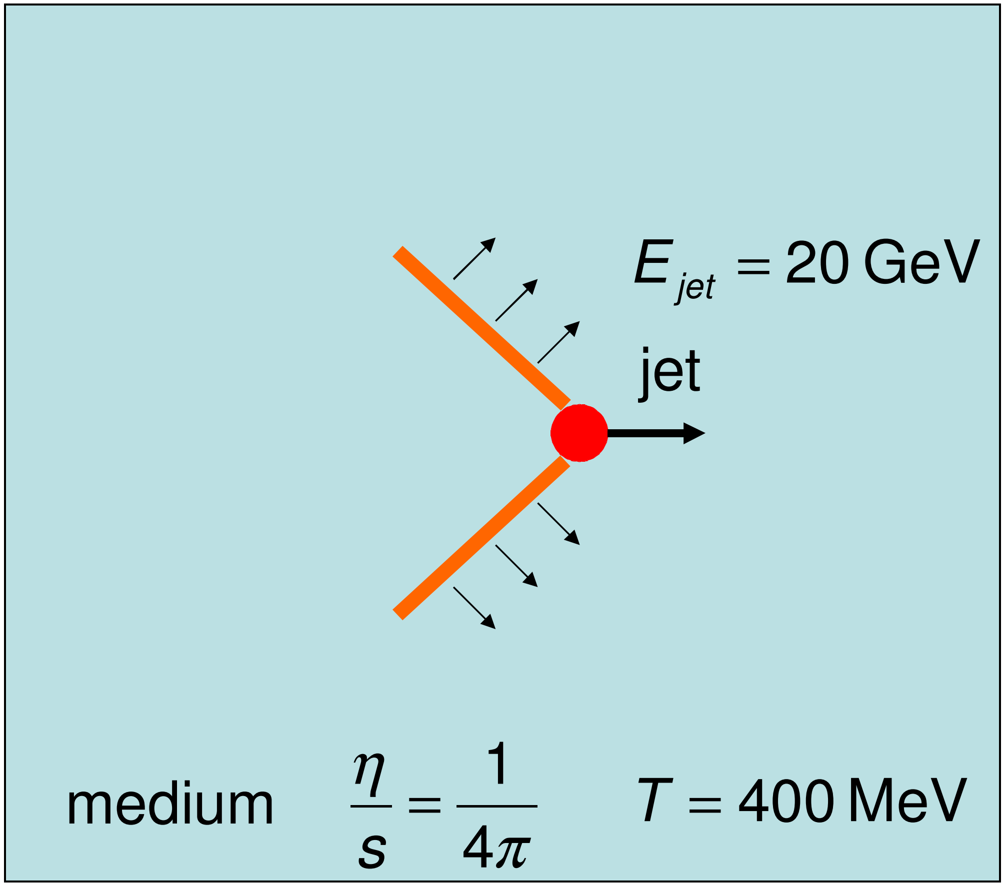


## Israel-Stewart equations from the kinetic theory ($\Pi = 0$)

$$
\begin{aligned}
\Delta^{\mu}_{\lambda} D q^{\lambda} &= -\frac{1}{\tau_q}\left[q^{\mu} + \kappa_q \frac{T^2 n}{e+p}\nabla^{\mu}\left(\frac{\mu}{T}\right)\right] - \frac{1}{2} q^{\mu}\left(\nabla_{\lambda} u^{\lambda} + D \ln \frac{\beta_1}{T}\right) \\
&\quad -\omega^{\mu\lambda} q_{\lambda} + A_1 \sigma^{\mu\lambda} q_{\lambda} - \frac{a_1}{\beta_1}\pi^{\lambda\mu} D u_{\lambda} + \frac{\alpha_1}{\beta_1}\left(\partial_{\lambda}\pi^{\lambda\mu} + u^{\mu}\pi^{\lambda\nu}\partial_{\lambda} u_{\nu}\right) \\
D\pi^{\langle\mu\nu\rangle} &= -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) - \frac{1}{2}\pi^{\mu\nu}\left(\nabla_{\lambda} u^{\lambda} + D\ln\frac{\beta_2}{T}\right) \\
&\quad +2\pi_{\lambda}{}^{\langle\mu}\omega^{\nu\rangle\lambda} - 2\pi_{\lambda}{}^{\langle\mu}\sigma^{\nu\rangle\lambda} - \frac{\alpha_1}{\beta_2}\nabla^{\langle\mu}q^{\nu\rangle} + \frac{a_1'}{\beta_2} q^{\langle\mu} D u^{\nu\rangle} \\
&\quad +A_2 q^{\langle\mu}\nabla^{\nu\rangle}\frac{\mu}{T}
\end{aligned}
$$

- All terms from the framework of Israel and Stewart included
- Equations without red terms refered as **'reduced' IS equation**
- Not yet complete IS (would need $O(\varepsilon^2)$ in $f(x,p)$)

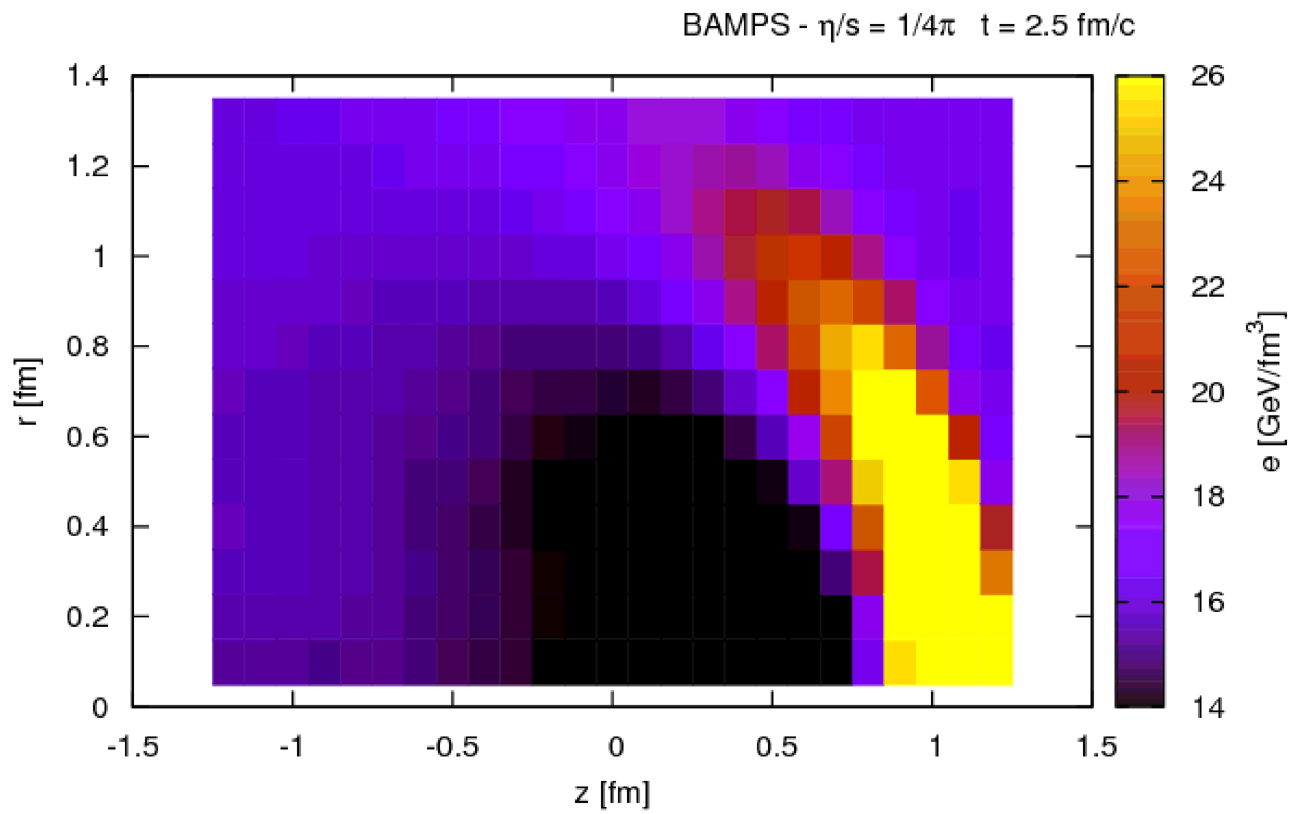
# 3. Mach Cone Formation




interactions: 2 –> 2 with isotropic distribution of the collision angle

## 3. Mach Cone Formation

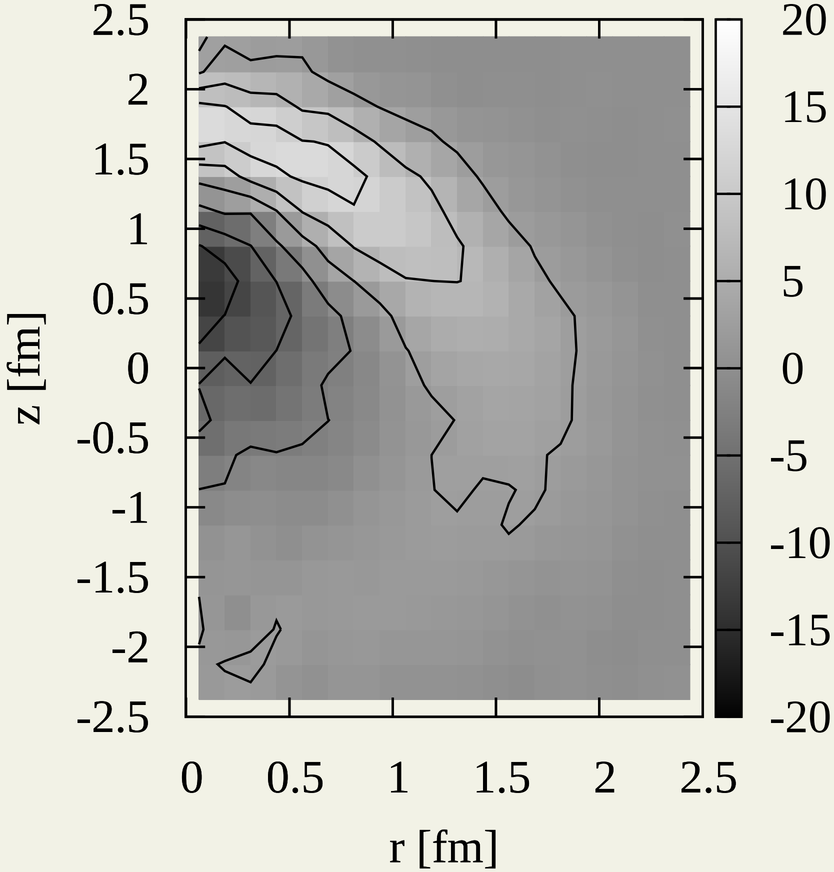
local energy density



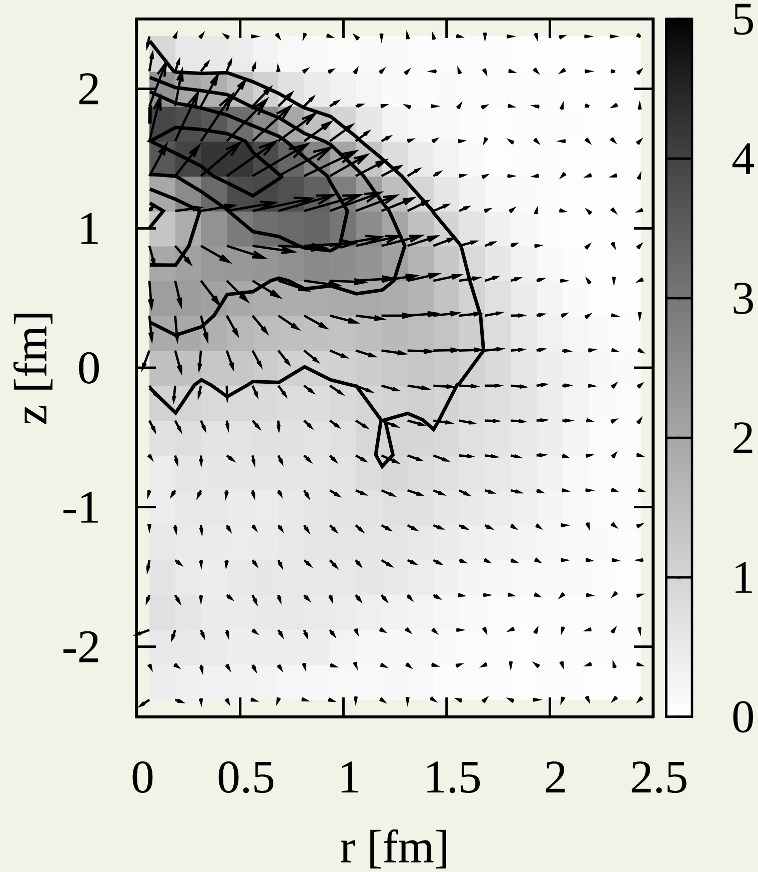

by I. Bouras, F. Lauciello et al.

## very similar to results from MPC DM, arXiv:0908.0299

### energy density



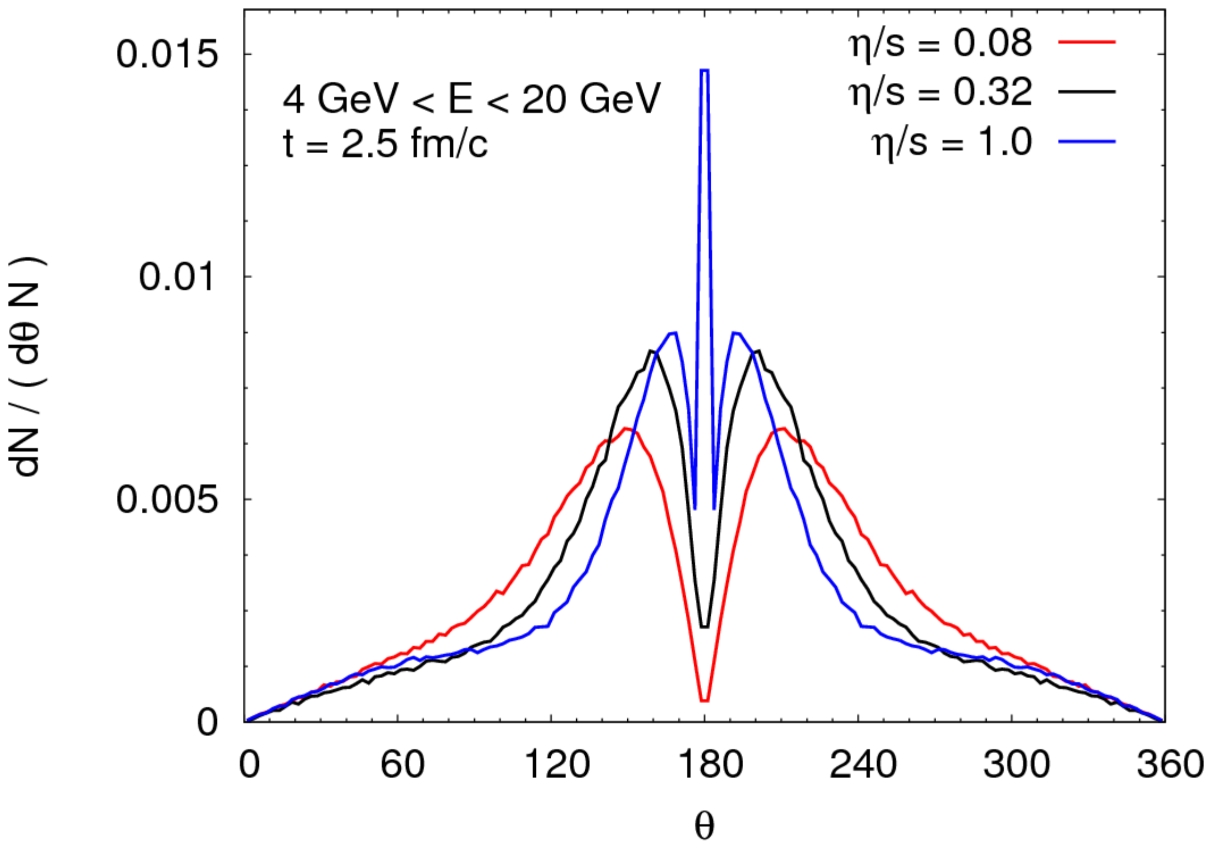

### momentum density




(isotropic $2 \rightarrow 2$ transport, $dE/dx = 68$ GeV/fm, $\lambda_{MFP} = 0.125$ fm, $T = 0.385$ GeV, $v = 0.9$)

Xu sensitive to viscosity

# 3. Mach Cone Formation



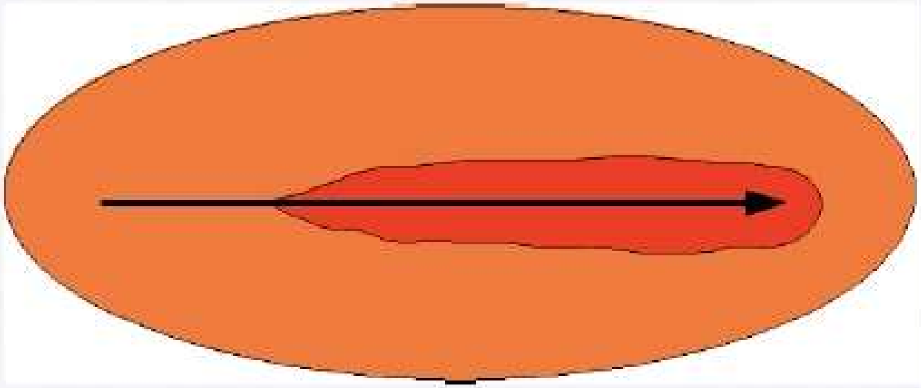

viscous effect: enlarge the mach angle

Comparisons with viscous hydro calculations will follow.

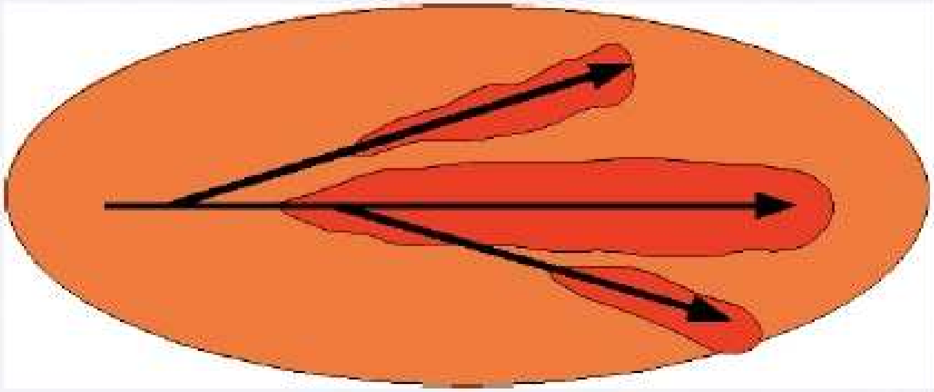
Zhe Xu

# How about multiple partons

A single parton deposits energy and transverse momentum in medium. This just $\hat{q}$ and $\hat{e}$



multiple radiation increases the sources of mom. dep. We know how much radiation as we already calculated it for energy loss



Can this be rigorously calculated ?

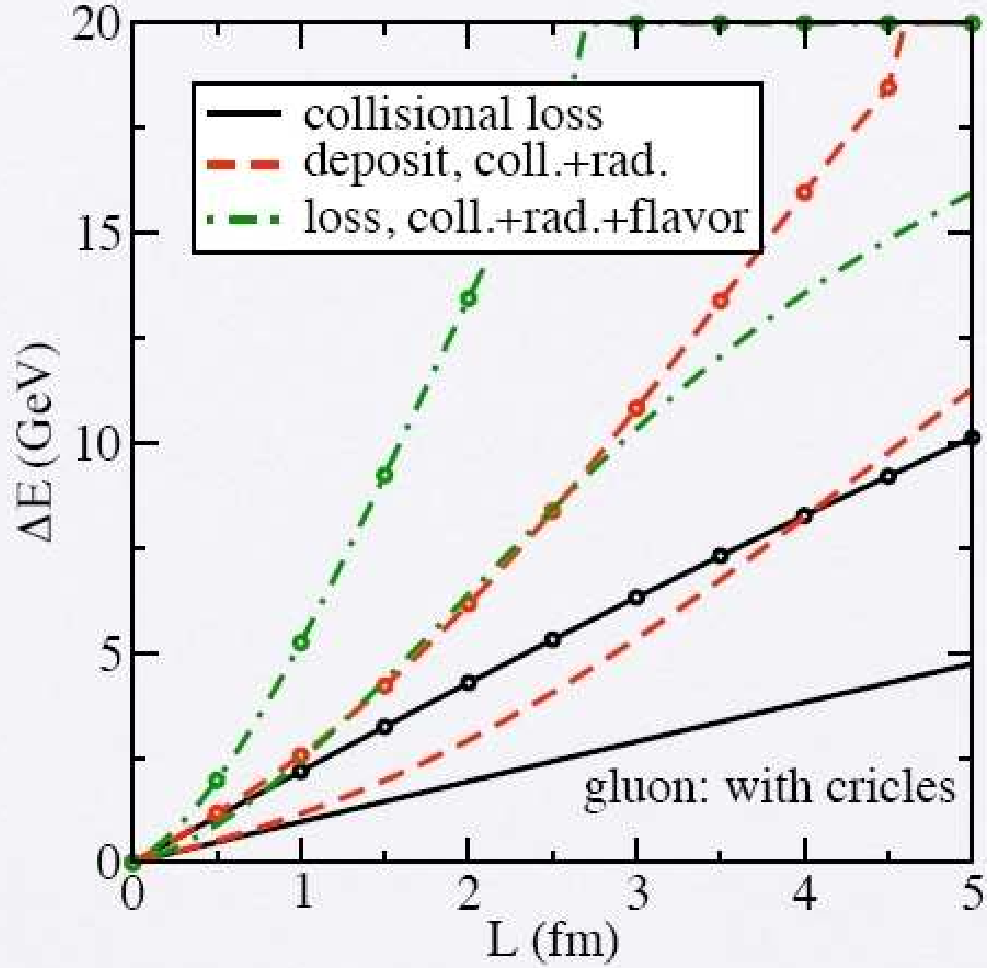
# The Results!

Need an input dE/dL

just for now,
use an HTL form

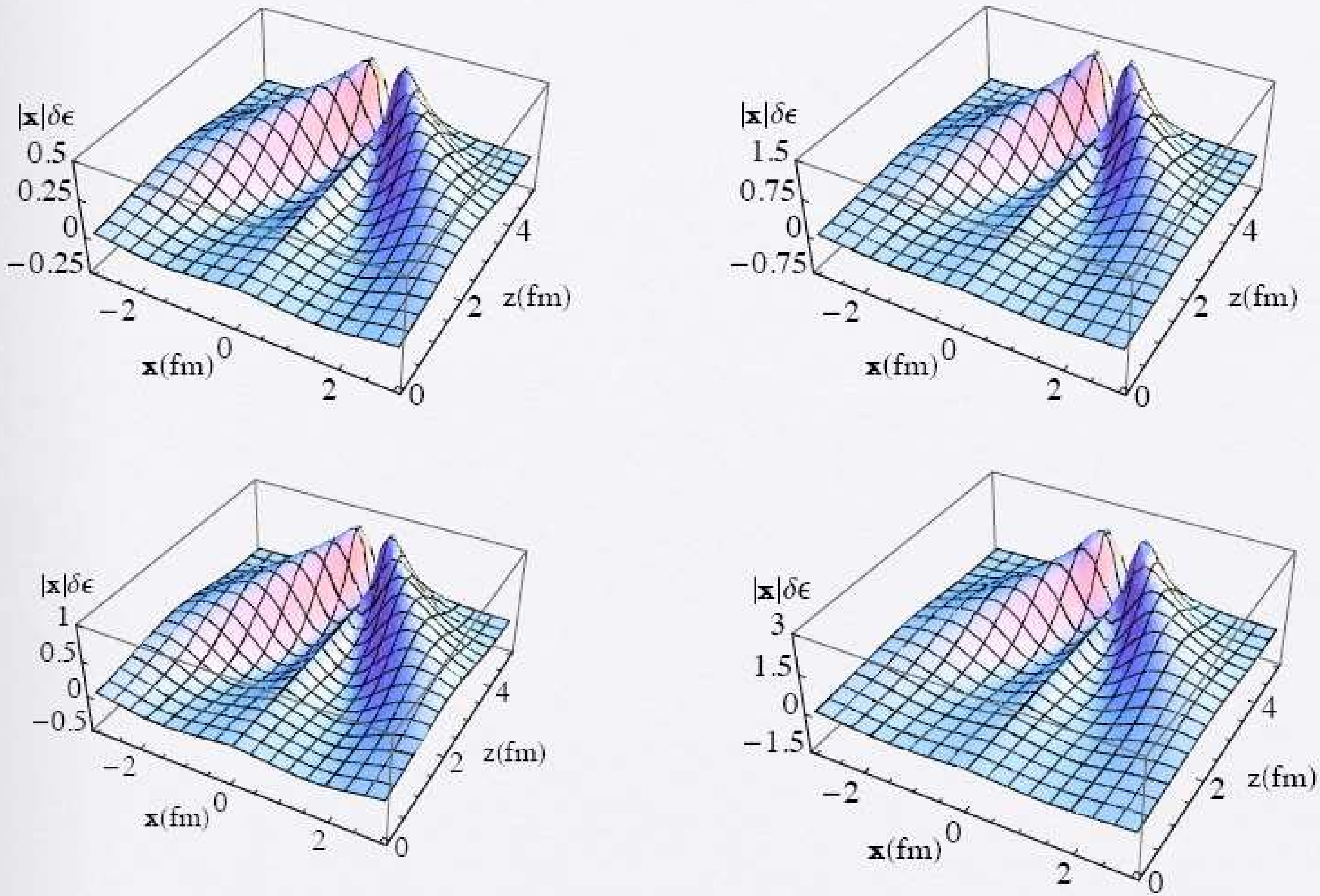
$$\frac{dE}{dL} = \frac{C_R \alpha_s m_D^2}{2} \log\left(\frac{\sqrt{4ET}}{m_D}\right)$$

T=300 MeV
E = 20 GeV




The energy deposition rate grows with distance
Only a part of total $E_{lost}$ is deposited!

# The results: II




Depositing the energy at the end enhances the Mach cone!

Neufeld very similar study

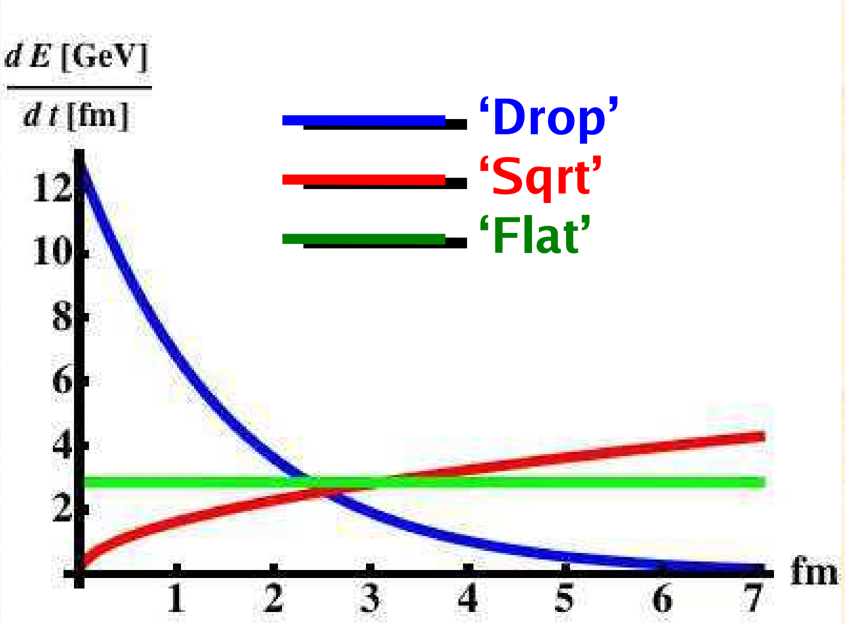
# Compare the emission spectrum for:

$$\lambda = 0,\ 0.25,\ 0.5,\ 0.75,\ 1.0\ \mathrm{fm}$$

$$\frac{\eta}{s} = 0.05,\ 0.10,\ 0.15$$

$$T = 250\ \mathrm{MeV},\ c_s = 0.57$$

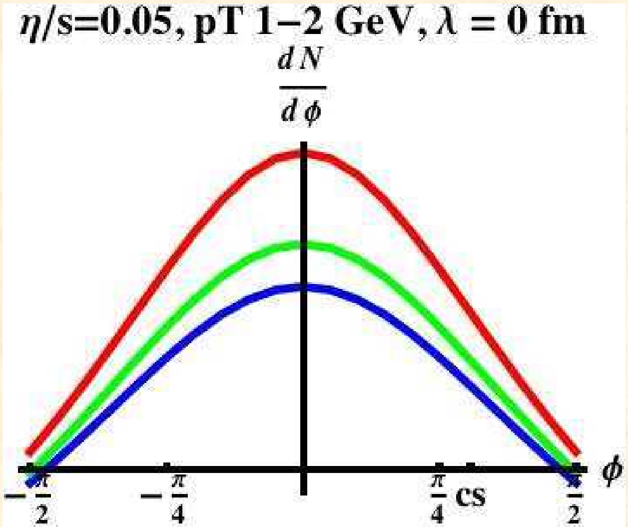
## And different forms for dE/dt:

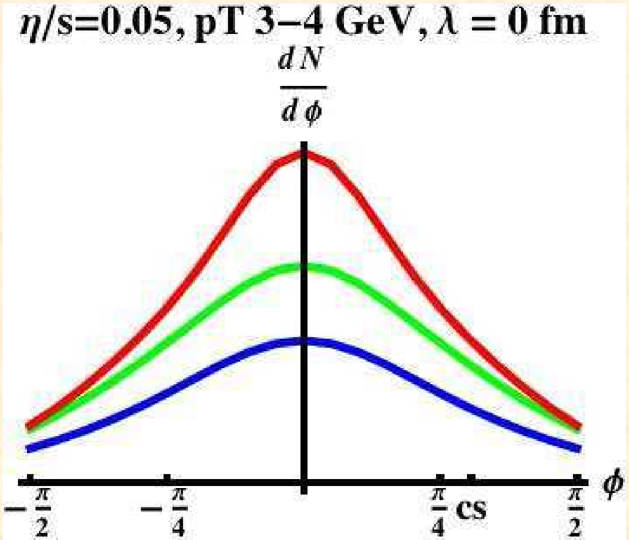


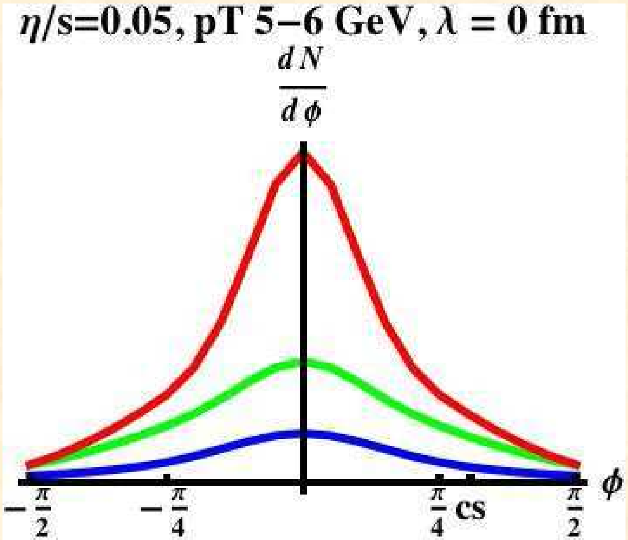

**In each case, the total energy deposited is 20 GeV, but each shape is motivated by different physics**

Neufeld "crescendo" gives strongest signal

# The spectrum for lambda = 0, eta/s = 0.05




qualitatively similar

even though $\sqrt{t}$ growth does not agree with Majumder's result...

# Summary

**Progress on several fronts:**

- matter properties (EOS, hadron gas transport coeffs)
- initial conditions (fluctuations)
- viscous hydro (results for bulk viscosity, tests of Israel-Stewart, revised formulations)
- viscous freezeout, mixtures
- Mach cones

**Still more work remains...**

**stay tuned for exciting new results at the next CATHIE-TECHQM meeting**

**most likely sometime Fall 2010**